

UNIVERSITY OF CAPE TOWN

FACULTY OF EDUCATION

TOWARDS EFFECTIVE CHILD-CENTRED MATHEMATICS EDUCATION

WITH LESSONS DESIGNED ROUND THE USE OF A FILM

A dissertation

presented in partial fulfilment

of the requirements for the Degree of

MASTER OF EDUCATION

by

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## A B S T R A C T

The crucial issues associated with child-centred learning in mathematics are seen to be addressed by the lessons and activities developed round the film. The lessons have captured the essentials of the humanistic mathematics method. Children become actively involved in discussion amongst themselves to the extent that they

- (a) think about mathematics;
- (b) contribute confidently;
- (c) make conjectures;
- (d) listen critically to one another;
- (e) feel suitably challenged to work together to prove their hypotheses for themselves,  
and, should they succeed,
- (f) are prepared to stand up and demonstrate the truth of them to the class.

The lessons, or similar ones, used in their classrooms should afford teachers who apply them an introduction to this child-centred approach to mathematics teaching and learning.

## A C K N O W L E D G M E N T

I am deeply grateful to my supervisor and course leader, Professor Chris Breen. His vision, dedication and love for people made the M.Ed. course the inspiring experience that it has been. Even this Dissertation with its many limitations manages to hint at the enthusiasm he was able to engender in me for mathematics education.

I am thankful to my wife for being so indispensable in her support.

The quality of typing bears testimony to the careful work of Mrs Indi Farmer. I thank her for it; her understanding and advice in the last hectic days I also appreciate very much.

## KEY TO ABBREVIATIONS

There are two video tapes denoted by V1 and V2.

Counter numbers follow these abbreviations to indicate the location approximately of a quotation transcribed from one of the tapes. The counter numbers seem to vary depending on the video recorder used so that numbers indicate only the approximate position of an extract of the transcription.

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## I N T R O D U C T I O N

A film forms the vehicle; the lessons designed round it are the paths; the setting is represented by the pupils in their classes; lastly, the direction and destination are supplied by philosophers, educationists, psychologists and other inspired people. They have contributed to the concept of the humanistic mathematics education and I name some of them because of their influence on me, and consequently on this dissertation.

The writings of David Wheeler, who ascribes his inspiration to the ideas of Caleb Gattegno, Mary Boole and others, have in turn inspired and guided me in the design and the teaching of the lessons, whose effects on pupils are described here.

The contributions to education of the aforementioned people have been truly inspiring so that my studies have had such effect on my life that I regard my work teaching mathematics to prospective primary school teachers and my witness as a Christian to have received a tremendous boost, by reason of new strong conviction and revived enthusiasm. My conviction concerns the power for effective learning of the humanistic mathematics method - learning that embraces meaningful understanding of mathematics and fulfilment of children as far as their confidence in themselves and the use of their mental powers are concerned. My enthusiasm is directed to affording the students that I teach the experience of learning mathematics this way and inspiring them

with its effects and instilling in them the skills that should enable them to adopt such a child-centred approach when they go out to teach.

Also, my enthusiasm extends to the teacher serving in the classroom. In the course of demonstrating these lessons to teachers in various schools, using classes that they provide, the interest and enthusiasm with which the lessons have been received has impressed me with the need for reform in mathematics education that teachers themselves feel exists. My intentions are to involve teachers in an in-service action research programme, where lessons similar to these ones are designed in a collaborative venture and taught in their classes. I plan to make myself available for personal contact with such teachers - visiting them in their schools if need be - and also to involve myself with them as groups, when discussion about their experiences ensues.

Along with strong conviction must come careful objective thought with the humility to change and adapt to suit the needs of pupils and teachers. As it purports to concern itself with children and their mental powers and with mathematics as a joyous, fulfilling exercise of the human mind, this dissertation can expect to be accorded attention. The unknown, untapped powers of the mind are the subject of discussion at present; mathematics, the object of awe and fear for most people, nevertheless claims the attention - be it in a negative sense - of the majority. When, as is claimed in this dissertation, the

two topics - children and mathematics - have encounters that bring to great advantage both mind powers and the joy inherent in mathematics, not only attention to, but application of the method described here can be expected.

I trust that, driven by the urgency of the situation of mathematics education in school and impressed by the forces of example described here, those concerned in bringing children and mathematics together in meaningful combination will find themselves impelled to apply exploratively the approach advocated in the pages that follow. What I trust represents some of the answers feature in these pages; an approach is suggested and its implementation described. Enthusiasm stems from the developments noted here and enthusiasm is one element needed if further developments are to accrue.

## CHAPTER ONE

### THE PROBLEM

#### 1.1 SCHOOL MATHEMATICS

##### 1.1.1 EFFECTS ON PEOPLE

Recounting the damaging effects of school mathematics on people is alarmingly easy to do. As Pearla Nesher notes, "The failure in learning maths is well documented and is in sharp contrast to the growing need in our technological society for adults who can handle mathematics fluently" (Wheeler, 1981, p.27), and from Banwell, Saunders and Tahta, also on the subject of school mathematics, "Most of the adult population seemed to be dramatically scarred by their experience of having to learn it." (1972, p.3)

Indeed the scene of disaster of mathematics education is everywhere to witness. Victims of the depredations of school mathematics feature in my own life. I refer to myself, my sons - the elder, shabbily equipped as he is with school mathematics, desperately trying to meet the demands of the 1st year Mathematics course of a degree in Mechanical Engineering, the second faced with the dilemma of developing mathematics skills - and enjoying it! - or bowing to the demands of his end-of-year school-leaving Matric' Certificate and resorting in the end to 'swotting' rules and formulae. I have nephews, numerous past pupils - some the products of my own teaching -

who suffer today because of the school mathematics experience. I know that were I to offer extra mathematics classes to pupils who are at present the suffering recipients of school mathematics, I would be hard pressed to do anything else but devote my time to teaching them how to cope with a subject that has become the most dreaded of all subjects that grace school timetables.

#### 1.1.2 TEACHING IT

The suffering deriving from school mathematics, and that has given rise to negative attitudes (Banwell, Saunders & Tahta, 1972, p.61), unfulfilled potential (Wheeler, 1975, p5), inability to cope (Hiebert, 1984, p.507), and powers that have atrophied (Hedger & Kent, 1980, pp.149-150) - ills affecting people in all walks of life - makes it imperative for all concerned with mathematics education to consider and apply new approaches in the learning of mathematics. The challenge is three-fold. It arises firstly out of enlightened theories about knowledge and about possibilities for the full utilization and enjoyment of powers of the mind unique as to degree, range and variety for each individual person. Secondly, it is about problems of the 'knowledge-explosion' (Wheeler, 1975, p.5), and this is the view of many educationists today, and mathematics, strange as it may seem to some, has a part to play, maintains Wheeler (1975, p.5). Of course, the view that it can play a part, implies that education was partly responsible for the problem. If it is

Wheeler's contention that, "however little it may be", it is "teachers in their classrooms in whatever circumstances they find themselves" that "can determine the impact of education upon children" (1975, p.5), then legitimately - since teachers and classrooms are not solely some modern phenomenon - it is they who have partly produced present conditions. Wheeler focuses on what mathematics in the classroom can do, and again teachers and classrooms concerned with mathematics have to bear part blame for conditions as they are. A look at the way mathematics is and has been at fault could point the way to a new approach, giving force to the argument that education in general can play a part.

On education in general, SYNCOM, an association in private enterprise dedicated to addressing problems in South Africa, in education, amongst others, has no doubts about the role education plays in society's ills. It states (1986):

"The crisis in education is global; so is the search for solutions. In the South African situation the main grievances are: lack of relevance, lack of educational choice, declining performance in spite of increased spending, and glaring inequalities between the races in infrastructure, teacher qualifications and examination results, to mention a few."

In the same paper SYNCOM deplores the waste that

"the indiscriminate linear increase in secondary education and matriculants, providing neither essential skills nor relevant and meaningful knowledge"

represents for South Africa. When Hiebert (1984, p.507) concludes that in mathematics education, research shows that success has been achieved in teaching 'form' in mathematics at the expense of 'understanding', so that pupils are able only

to "memorise and follow rules" (p.507) without making sense of them, and consequently without being able to apply them, mathematics then can be seen to be party to the production of a body of irrelevant and meaningless knowledge.

### 1.1.3 MATHEMATICS AND KNOWLEDGE

Perhaps it is what we regard as 'maths knowledge' that is at fault. Giroux states, "... knowledge is often treated as an external body of information, the product of which appears to be independent of human beings" (1981, p.52). Textbooks perpetuate the lie, presenting, in Marshall Gordon's words, a "finely polished, impersonal mathematics curriculum" (1978, p.252). Teachers, then become 'presenters' of this knowledge and pupils the 'recipients', and a process of degeneration perpetuated. 'Presentation' begs some quality so that questions become either criticism of presentation, with the attendant slights to the presenter, or worse - so far as the pupil is concerned - indication of deficiency of intellect on the part of the recipient. So, despite repeated injunctions to "ask if you don't understand", nobody asks questions. Dialogue between the teacher and pupil is reduced to what Ian Thompson, in an article in the Times Educational Supplement, refers to as the "usual monologue" with the pupil at the "receiving end" where "the major form of oral response given by children is of the 'guess what's in teacher's head' variety" (1986, p.41). Understanding mathematics becomes, then, the preserve of the brilliant, with ordinary pupils resorting to memory to see them through as far as it can, and we arrive at the 'irrelevant-and-meaningless-knowledge' stage.

Or are there really so few people capable of doing mathematics? Margaret Donaldson, in her book, "Children's Minds" (1979, p.85) answers the question with reference to disembedded thought (whether only a few people are able to learn to cope with abstractions or handle systems) with the categorical statement that "there is no reason to suppose that most of us - or any of us for that matter - manage to come close to realising what we are capable of", and she regards even the idea of an upper limit to be irrelevant because no one has come close to it (p.85). Brief reference to views of Piaget on the subject of knowledge, as expressed by Margaret Donaldson in an appendix in her book, "Children's Minds" (1979, p.129), are pertinent. She writes (p.140) :

"Piaget tells us again and again that knowledge does not come to us from outside, 'ready-made'. It is not a 'copy' of reality - not just a matter of receiving impressions, as if our minds were photographic plates. Nor is knowledge something we are born with. We must construct it. We do this slowly, over many years."

The reference to persistence to communicate on Piaget's part indicates that our view of knowledge has needed correction. We do seem to have missed the boat. P.C. Dodwell, writing on 'The Spatial Concepts of the Child', queries why, if certain fundamental notions demonstrated by Piaget have to be developed by the child before it begins to understand the ideas of Euclidean geometry, it took so long for education to take cognisance of it (1959, p.6). He concludes (p.7) :



"The answer, I suppose, must be that no one has thought very carefully about the genesis of factual knowledge in this way before. On the one hand philosophers, worried by problems of epistemology, have been content to analyse concepts about space, time and so on, without bothering to find out whether the actual genesis of these concepts in young children could throw any light on their philosophical arguments; on the other hand teachers have, I think, tended to accept too easily traditional ideas about the way teaching should be promulgated, or have assimilated too freely ideas fashionable in some current psychological school without being too careful to observe whether the child's actual behaviour and development can be understood with the aid of such ideas."

Traditional lecture-type lessons have pandered to the teacher's preoccupation with always being right. Perhaps it is the reluctance to present pupils with the sight of an error, either for the child's sake or because of the reflection cast upon the teacher's ability. We tend to ape the textbooks by seeing to it that our pupils never witness Teacher in a position where he or she is searching for a solution. We have been reduced to act like small-time performers, ever watchful that we are very well acquainted with whatever it is we are presenting, or sufficiently rehearsed beforehand in every step of the procedure to be taught, so that our unenlightened - and unsuspecting? - pupils never see us at a loss or searching for what to do next. David Kent and Keith Hedger comment on this tendency, as follows : "Beyond the normal barrier of reservation, mathematics teachers appear to think they have a vested interest in 'not being seen to be wrong'" (1980, p.144). There is, of course, the pressure that examinations exert on our

teaching.. The quantity of mathematics - 'service' or 'user' - that the different disciplines require for their calculations, which mathematics teachers have felt obliged to teach, and which the biologists, physicists, geographers and the like are quite competent to teach their pupils to apply, places excessive burden on mathematics teachers.

Whatever the cause for school mathematics being taught the way it is, it is children who in the future will have to occupy the centre of attention in the thought and practice of teaching mathematics. We are going to have to take into account children's thinking. The emphasis will have to be on the powers children have for learning. While David Wheeler concedes that not every child can be expected "to find mathematics an engrossing study" to the extent that he or she wants to pursue it as the main subject in school - or out of it - he does insist that the opportunity to experience the "power and excitement of mathematics" should not be denied any child (1975, p.4). We cannot afford to propagate 'product' above 'process'. Actual mathematics activity that starts usually with ill-defined problems and revision and re-thinking before it gets under way, must feature in classroom 'maths'.

"This fitting and re-fitting is rarely mentioned in mathematical literature today, though some of the great classic papers are full of subjective accounts of the mental paths which led to the final result."

(Banwell, Saunders, Tahta, 1972, p.6)

Delaney (1982, p.20) explains, "Euclid's systematisation was so powerful ... that deductive mathematics became the only

acceptable mathematics (or the only thing acceptable as mathematics) ...", with the consequence that "little or no attention has been paid to the origins of mathematics, neither in the originating problem situations that may arise nor in the generating activities of individuals that develop the tools to solve the problems ..."

Group work where children have "opportunity to share mathematical ideas and to discuss problem-solving approaches with other children", states Thompson on schools in the United Kingdom, in an article in the Times Educational Supplement, "are reported by H.M.I. to occur 'in only about 1 per cent of the secondary classrooms' they visited", and he adds that reports of researchers are that often where children are encountered in groups they are "working on individually assigned tasks rather than working collaboratively on a common or shared task" (1986, p.41).

#### 1.1.4 OBSTACLES TO CHANGE

It is true that teachers have obstructed change. Their preoccupation with a style "based on exposition, consolidation and practice" - one that shows them "judged by examination results" to "have been successful in their teaching" (Pirie, 1986, p.Intro.1) - has presented the major obstacle to change and progress in teaching approach. However, resistance is not all from one quarter; others share the blame with teachers. That teachers have to contend with the objections

and resistance from children and parents - the very beneficiaries of the changes they might venture to implement - is perhaps the most disconcerting factor to protagonists of change of approach in mathematics education. Not only does such resistance come from a wide front, its vehemence is quite unexpected and appears to be quite unreasonable, directed as it is at teachers intending only good for their pupils.

It is necessary at this stage to reflect on some of the causes of resistance occasioned by changes in teaching approach in mathematics. Dr Susan Pirie (1986, Appendix Intro) lists the following 'fears and reservations' of teachers who consider the investigatory approach :

I shall lose control of whether they are doing worthwhile mathematics.

I shall lose control of the class if they talk and wander about.

It is bound to be noisy and disturb others.

It will involve too much furniture shifting and apparatus.

I have not time because of exam and syllabus pressures, and so

- it is only for the lower ability range;

- it is only for 1st and 2nd year pupils.

My classes are too large and we have no equivalent of the laboratory technician.

The pupils won't like it because it "isn't real maths".

My role will be eroded; I won't be teaching them.

Exposition and practice have always worked for me.

I may not be able to cope with some of their questions.

The pupils will be all over the place with their maths.

How can I know what they have learned?

The above statements are revealing, pinpointing the cause of reluctance to change, as stemming from teachers' attitudes because these are remarks of teachers. They help to bring to light other very real forces at work on teachers. It is clear that much adamancy against change stems from teachers' own egotisms, desires to stay 'comfortable', anxieties ("I won't be really teaching them" and "I may not be able to cope with some of their questions"), problems of assessment and attendant evils, and resistance of fellow-teachers. However, the opinion of pupil and parent also focuses on teacher as target, exerting tremendous inhibiting pressure that cautions one to 'play safe' when it comes to changing methods of teaching.

It is the latter relatively less known factor - the opinion of people in general concerning mathematics - that deserves scrutiny. A number of causes contribute to it.

It seems traditional teaching has worked too well. The traditional view of the school's function as one of 'passing on knowledge' seems to have permanently embedded itself in people's minds; they see mathematics - and all subjects for that matter - as knowledge that 'experts' have to give them. 'Cookery book maths' refers to the process that panders to this view. The 'ingredients' - mathematics terms, etc. - and

the 'recipe' - formulae and rules - are presented to the pupils who are then to 'make the cake' by producing the answer. Understanding and real mathematical action go by the board in this system, so that 'thinking' deteriorates into remembering or finding out, in whatever way one can - usually quite independently of any understanding of the mathematics - what the next step production of the answer entails. As understanding loses way the demand for clear prescriptive rules intensifies, and with it fears and insecurity grow. Experiences of mathematics become traumatic ones. A vicious negative spiral of loss of confidence in oneself and mindless attachment to rules ensues. Pressure on the teacher to comply with demands of pupil and anxious parents intensifies. 'Teacher telling' (A.T.M., 1984, p.5), consequently, takes over as the 'activity' in the classroom, ensuring perpetuation of the whole sorry business.

Preoccupation with mindless mathematics seems to be infectious and addictive. I encounter strong opposition to my efforts to present action mathematics that involves pupil or student. Fear of being 'confused' is the reason given. Does thinking for oneself incur responsibilities that one feels fearful about assuming? Does development of one's own abilities present possibilities that one is reluctant to take on, because one might be landed in situations beyond one's capabilities?

The importance accorded mathematics in the world of commerce and technology today contributes to the nature and force of the opinion. Satisfactory examination results in mathematics are essential for entrance to careers, it is assumed, and so "... teachers feel under pressure from the syllabus and the need to help students achieve high grades" (A.T.M., 1984, p.5).

## 1.2 SHELIVING THE ANSWER

Agitation for change in the teaching approach of mathematics started many years ago. The writings of - to name just a few - Mary Boole, Caleb Gattegno, Donaldson, Skemp, Gordon, Giroux, Wheeler and Peter Kaner, many of them influenced by findings of psychologists and educationists like Piaget, have drawn attention to the processes of children's minds for guidance about teaching them mathematics. Their efforts have not been in vain. Reform in mathematics teaching, though long overdue, is materialising. In the United Kingdom changes in attitudes to evaluation and assessment have caused significant departures from traditional examination procedure, with a consequent releasing of much that inhibited teaching practice.

Such reforms focus attention on teachers. The call is for new approaches to teaching. Who other than teachers need co-opting as the agents of reform in learning in schools? Dr Susan Pirie, in an article in the Times Educational Supplement, called "Your Investigations", pinpoints the crucial issue when she describes the

difficult changes teachers have to undergo to teach in this way. Many educationists regard the changes in teaching as the central problem of reform in mathematics teaching, according to Pirie (1986, p.42). They see no scarcity of ideas for reform in mathematics education but are frustrated by the non-implementation of them in the areas where they are so desperately needed.

Good ideas calling for reform in teaching are worse than useless when expressed in statements in theses and research treatises that languish on shelves in university libraries. When this is their lot, they condemn themselves as part cause of the conditions they inveigh against and they stand as indictments of research procedure in general. Research - in particular, research in education - that calls for change and action is obliged to address the issue of reaching the agents that implement such proposals. The impact of ideas on reforms in teaching must be felt in the classroom and preparing research so that it is palatable for use by teachers in classrooms should just be assumed without question to be part of the job of research.



## CHAPTER TWO

### AN ANSWER

#### 2.1 INTRODUCTION

With reference to a "substantial illiteracy, widespread incompetence and incomprehension" that obtains "in all areas" of mathematics, David Wheeler (1970, p.23) exonerates attitudes, aims, curricula, the actual physical provision for schooling, like surroundings, buildings, classroom conditions and teaching materials in his search for causes of the lack of progress in mathematics education and he focuses on "actions of the teacher" as the culpable factor.

Basing his arguments on the theories of knowledge and of the learning of mathematics, referred to in Chapter 1, Wheeler argues for a 'scientific method' of teaching and method of study, where observation of the pupil is guided by 'a sense of truth' reminiscent of Gattegno that is independent of the number of people in agreement or disagreement with the pupil or of his or her personal abilities or opinions. "We are not less sure that the earth is round because there are some people still alive who believe that it is flat" (Wheeler, 1970, p.25). The 'sense of truth' can apply to those same facts for others who draw their conclusions in obedience to the rules of the discipline (Wheeler, 1970, p.25). True to aims that he maintains we virtually all believe should be achieved, Wheeler explains that 'scientific' teaching concerns the powers of the child and their development - child-centred learning - so that while teacher is effacing himself or herself, the child receives attention, and it is about the child that activities in the class revolve.

Wheeler calls the teaching he advocates scientific because the scientific method is one of observation, where the principle technique of the scientist is to act upon one's observations. Humanistic mathematics education follows this principle but must not be confused with any particular teaching method. It is essentially an attitude born of awareness and appreciation of the child as a created being possessing breathtaking powers.

Skill and competency, self-confidence and originality that contribute to the child's 'autonomy', Wheeler maintains, and a 'self-awareness' that constitutes the 'sense of autonomy', together make up the aims of the scientific method (1970, p.26).

Encompassed in the mathematizing that characterises the approach Wheeler advocates are activities of investigation and discovery, using class and group discussion, individual and corporate work with manipulative materials and other related activities and these stem from the theories of epistemology and cognitive development of Piaget and his followers and interpreters.

Margaret Donaldson, describing Piaget's three main stages of cognitive development, in "Children's Minds", mentions Piaget's repeated denials that knowledge as a representation of reality has to be 'received' (1979, p.138). She points out that 'internalisation' that Piaget refers to, and that succeeds the practical representation of things, during the sensori-motor period, is an 'act of thought' of the concrete operational period (p.138). Donaldson draws attention to the term 'concrete', underlining that the child's thinking at this

stage still concerns "doing things with physical objects" (p.138). Postulating, reasoning logically and drawing conclusions constitute the thinking of the formal operational period, and as Donaldson explains, such mathematical reasoning is characteristic of science - the planning of experiments and steps to "formulate general rules" based on them (p.139).

The fear of the teacher that standards might drop with the new emphasis on participation in discussion and activity that the lessons imply is a real one. Mastery of "formal systems of thought such as mathematics", according to Margaret Donaldson (1979, p.82), and all formal or abstract thought denotes ability to think "beyond the bounds of human sense", operating so that it is "not within the supportive context of meaningful events" (p.76). Donaldson calls thinking that is formal 'disembedded thought' and points out: "The better you are at tackling problems without having to be sustained by human sense the more likely you are to succeed in our educational system ..." (pp.77-78). The highest value is placed on symbolic thought, where  $p$  and  $q$  replace familiar objects, and "when it comes to  $ps$  and  $qs$ ", states Donaldson, "most of us have to struggle" (p.77), adding, "The human mind does not engage easily in the manipulation of meaningless symbols" (p.77). Pointing out that practical disciplines like engineering would not exist were we to "abandon the arduous task of functioning without the support of the world of familiar events" (p.82), Donaldson emphasises the necessity of considering "the structure of things", "manipulating systems" and "abstracting forms and patterns". She concludes that to discard the acquiring of such skills in education would not be the answer, but rather to alter the disproportionate emphasis accorded "intellectual skills" by

"increasing the value we place on other things" (p.83). For example, in engineering, the use of "a lathe or a milling machine" - which Donaldson deplores university departments of mechanical engineering neglect to teach - should be accorded more weight. Adding emphasis to her proposal, Donaldson quotes Alfred North Whitehead (p.83) in support of the statement that disembodied thought actually "yields its greatest riches when conjoined with doing".

## 2.2 MATHEMATICS AND MENTAL POWERS

'Doing maths' implies working at a topic in much the same way it was originally approached when the mathematician first came upon it. In the investigative or discovery method children are led to work at mathematics in this way, so that what is of relevance for them are the powers and activities employed by the mathematicians that produced the mathematics that we encounter today. As the painting 'diaries' of great artists reveal the stages and the 'sweat' that went into the production of famous works of art, so mathematics education needs, according to Wheeler, "this accretion of results as it is, warts and all, so that learners might gain a sense of a human activity with all its admirable and foolish qualities" (Wheeler, 1975, p.6).

If children are to explore mathematical concepts on their own, they have to bring their powers to bear on topics. These constitute generally, taking more responsibility for their own learning and making more decisions on how they should work and in what direction. More specifically for mathematics, Gattegno cites, making transformations, noticing differences and similarities, using imagery, and, of course, employing memory (1971, p.25). Custom, textbooks and

training have conspired to make memory the one overworked power that, according to Gattegno (p.25), pupils bring to bear on the mathematics they do. Group discussion and activity round the viewing of the film serve to resuscitate neglected powers of "perceiving, stressing and ignoring, generalising, verbalising, testing and expressing in proper and adequate terms what has been found ..." (Gattegno, 1981, p.21), and to place such powers back in the learning process.

### 2.2.1 USING LANGUAGE

Film triggers images and stimulates activities that revolve about its topic, at the same time evoking the language that describes its contents. As Nancy Shuller notes, when relating her experiences while presenting a Nicolet film to her class:

"The mathematics of this film will belong to the learner because he/she had recalled its images, related the story of those images and led himself/herself to an understanding of the maths within." (1983, p.38)

It is thrilling, as shall be seen later, to hear children express mathematical items and concepts in their terms, because they are laying claim to that mathematics; even when terms are a little loose, others use more precise terminology, as in the case of the Std 6A's, as described in Chapter 4, when one pupil's description that "all the figures could fit into one block", was refined by the statement of another that "all the figures' areas are the same". Pupils are hearing one another and how much are they not learning from one another!

### 2.2.2 BEING CRITICAL : LISTENING, OBSERVING AND EVALUATING

Investigation presupposes the power to assess, a skill which is neglected by the kind of teaching that calls upon the teacher to act as chief judge and arbitrator of every statement uttered in the classroom. The habit of repeating every 'correct' statement and correcting every 'incorrect' one effectively switches pupils out of assessing their colleagues' contributions even to the extent of their hearing what they have said. In humanistic mathematics teaching the ball is put back in the court of the pupils every time comment is forthcoming, so that not only are they left to arbitrate on accuracy of the impressions of one another but they are themselves querying statements, calling for refinement of language, definitions of terms and learning to listen discriminately.

### 2.3 TEACHER'S ROLE

How do teachers, accustomed to the passing on of mathematical knowledge become facilitators of children's mathematical activity? What does it entail for the teacher, who has to organise a point of entry that draws the attention, invites the interest and encourages the involvement of every pupil in the class in activities that call forth skills and that cause encounters with facts they need to know? There is no one method; it is an attitude on the part of the teacher.

Pupils and their powers form the basis of his or her approach in lessons, and these serve as the touchstone in the formulation of all the steps of those lessons. Knowledge becomes incidental to the

process of arriving at it, 'vetting' it and accepting or rejecting it, so that Teacher takes a back seat as far as the role of Expert, Instructor and Sole-Supplier-of-Knowledge is concerned.

Writing in the Times Educational Supplement (10 October 1986), of the implications for the teacher, Dr Susan Pirie states :

"The personal cost of an attempt to approach pupil-learning in a novel way must not be underestimated. An act of faith is involved in the resolution 'I'll stop telling them everything and let them discover mathematics for themselves' - faith in pupils' inquisitiveness, faith in one's own class management, faith in the gospel that mathematics can be learned in this way." (p.42)

Ian Thompson (1986, p.42) has this cautionary note :

"Historically, those innovations which have demanded a big change in the teacher's role - usually from 'teacher as fount of all knowledge' to 'teacher as enabler, facilitator or fellow-learner' - have produced a loss of confidence and an increase in anxiety amongst teachers."

The need for teachers to be agents in humanising mathematics education and the urgency to bring about this change, persuades Wheeler that it is, "what can be done, however little it may be, by teachers in their classrooms in whatever circumstances they find themselves", that is worth attention, adding, "The social circumstances need changing, for sure, but it is still the classroom that can determine the impact of education upon children and that cannot wait until the whole world is straight (1975, p.5).

The fund of ideas generated by the film and stimulated by group discussion and activities offers exciting fields of learning to traverse. But pupils need to actually gain entry to these fields if

more than just superficial learning is to be achieved. As Wheeler puts it, "... our exploration has turned up a number of important awarenesses" (1975, p.8), and he emphasises the need for these to be worked into a definition. The teacher's role here is significant and intriguing, because he or she is committed to preserving the freedom of thought and speech the children are enjoying, seeking also to endue them with a sense of responsibility for their learning. How does one lead children without forcing them, bringing them to the point where one knows true, profound learning experiences will be encountered? The fact that children are now contributing ideas encourages the scientific approach in a way that was denied, when Teacher made all the statements - as expert - thereby removing the necessity of proving any of them. On the other hand, pupils' statements - no matter from whom they come - are not accepted unconditionally, so that acceptance of the need to prove what is said is almost automatic.

Skill is needed in knowing what questions or suggestions to make in order, for example, to clarify language that is being used or to point a group into a specific area of investigation, without imposing one's own answers.

The teacher's task is to see to it that children's powers are activated and to facilitate their applying them to the mathematics at hand. Certainly, if the film, lessons and teacher have the pupils in a position to "witness what is happening as it happens" (Gattegno, 1981, p.21), and the climate is such that they want to find out why it has happened, the teacher can feel satisfied that they constitute very favourable circumstances for the development of mathematical understanding of the pupils. Gattegno (1981, p.21) states that



vitaly important is "the learning that a dialogue is possible between a dynamic situation (presented here by film) and one's mind".

#### 2.3.1 CLASS DISCUSSION

Alertness is the priority, as the teacher's role, in Wheeler's description of humanistic mathematics education, "is quite different from the teacher's way in conventional classrooms" (1970, p.27). In Wheeler's words, the teacher "must use every means he can find to focus the attention of the children on the problem, and this means that he must efface himself from their attention (1970, p.27). Meanwhile, as Wheeler points out, the teacher has "the children at the centre of his attention" (1970, p.27), because he is negotiating the transfer of the responsibility of getting the work done over to them, ensuring at the same time, as far as possible, that they are doing work that is relevant as far as syllabus prescriptions are concerned. This entails listening and thinking on one's feet, so to speak.

#### 2.3.2 GROUP WORK

Calling 'living' mathematics a 'communal possession' in contrast to 'dead' mathematics that exists in an atmosphere of competition, Mary Boole (1972, p.20) recommends the 'essentially communal' (p.21) method of working in groups as conducive to differentiation in children's learning of mathematics.

Mathematising or the humanistic mathematics approach is essentially a doing mathematics. Children become active learners when they engage in this sort of mathematical activity. We are out to teach them to seek, and therefore they have to ask the questions, and they have to answer them. This implies the added, new responsibility on their part to choose questions. Traditional mathematics teaching has teachers choosing the questions and asking them, with children trying to grasp in whatever way they can what is expected of them. In the 'film' lesson the introductory class discussion round the first shared viewing sets out the new approach. Once pupils realise that they "hold the key to their learning" (Shuller, 1983, p.41), group work becomes a very natural 'next step' in the learning process.

Group work follows class discussion, allowing groups to discuss and, with the help of manipulative materials, to express in concrete form what they have seen on film. Group work is accepted as an integral part of lesson activity or proceedings, from the start. In fact, all other activities centre round group activity, both viewing of the film and class discussion forming the basis of group work, with film providing the content for discussion and investigation, and class discussion serving to start pupils in thinking along the lines they are to pursue. It is in the group context that the investigative-type mathematics comes into its own. Children are encouraged to make suggestions and share ideas with one another and to consider critically what has been mooted. Groups are conducive to differentiation, children

being able to pursue what they want to, within reason, and delve into a topic to the degree of difficulty that suits them. It is important that children appreciate that they are free to investigate as they choose and that their investigations are valid according to what it is they perceive the film to be asking of them. In other words, any questions they ask are the 'correct' ones in their group's search, so that the paralysis inflicted by the fear of asking the 'wrong' question is eliminated.

## CHAPTER THREE

### THE PROJECT

#### 3.1 THE SHAPES' FILM

A lesson series introducing a method of teaching geometry by 'dynamic patterns', presenting geometric facts by way of a film, and by means of a child-centred approach that fosters creative and independent work, as discussed above, could be the answer. Lessons speak directly to the teacher in the classroom in terms of the task at hand, and when they are designed with the teacher in mind, for use by that teacher and structured to accommodate the teacher's adaptations to a particular class, a collaborative research action is achieved. A type of do-it-yourself demonstration situation is created, with the possibility of immediate feedback and evaluation, that should appeal to the practising teacher.

Lessons have been formulated, using a film, designed by Chris Breen and made by Christa Dietterle, showing change of shape and perimeter of certain kinds of quadrilaterals, while area remains constant. They are of a child-centred nature and should serve, when applied by practising teachers to their classes, to guide them in this method of teaching.

Avoiding prescribing a 'method', the lesson on dynamic geometry, using a film, insists only that the approach be child-centred. Gattegno, in the note, "A Few Suggestions for Teachers", in his book, "Animated Geometry" (1981, p.20), states that his intention is "not to make suggestions for others to use", because what guides him are events in

"my classroom with my particular students" (p.20), with the proviso, "so long as the yield is fruitful ..." (p.21).

Consequently the teacher, guided by general principles usually best fulfilled using group and class discussion with Teacher adopting a listener's stance and acting the role of 'chairman' instead of 'expert', aims for development of powers of the child. He or she organises learning in the class so that the pupils "produce much more than a uniform list of the geometrical facts perceivable to a traditional student of traditional geometry" (Gattegno, 1981, p.21). Skills of observing and of listening so as to "delve more deeply behind the scene and extract what they can of what is hidden", of the ability to "formulate synthetically" the various contributions of the class, and of the assurance that "what they perceive is being expressed by them in a manner that leads others to perceive it too" (Gattegno, 1981, p.21), contribute significantly to the development of autonomy.

### 3.2 LOCATION AND EQUIPMENT

Exploratory lessons on the Shapes' film were taught at the Simon's Town High School and at the Open Day of the Education Faculty's Mathematics Department at the University of Cape Town.

I taught four lessons to each of two Std 6 classes - an Afrikaans and an English one - at the Simon's Town High School over three days, from the 6th to the 8th of October, choosing the school because I had taught there for 18 years, leaving it to assume a post at the Cape Town Teachers' College in 1981. Knowing, consequently, many of the

staff, including the principal and secretaries, matters of organisation were facilitated, even down to parking, locating classrooms and so on.

Quite significantly, different conditions obtained when the film and lessons were presented at the Mathematics Open Day event to a 'class' of visitors. From a variety of schools catering for different culture groups, these 'pupils' ranged from Standards 6 and 7 to Standards 9 and 10 and had come ostensibly to view and to be entertained; to be put into a classroom situation and undergo a school-mathematics lesson was the furthest from their expectations or desires! Conditions like these afforded all the more reason for satisfaction at the performance of the young people who, when others scented possible challenge and work and absented themselves from the scene, stayed and - as we shall witness - were involved to the degree that they produced competent demonstrations of mathematical proofs devised entirely on their own.

Sue Brunditt was to operate the video camera to record proceedings in all the classes. She proved indispensable when it came to the organising and operating of all audiovisual equipment, and her helpful suggestions about actual teaching procedure, made from her viewpoint as observer, I came to value.

### 3.3.1 COMPOSITION OF CLASSES

The two classes of about twenty-five pupils each were, like average classes, finding mathematics a difficult subject, many pupils of Std 6B, the English class, lacking confidence in their ability in mathematics, and consequently, according to

their teacher, not enjoying their lessons. Just as the composition of the classes suited me, so did the classroom settings. Both classes occupied a lecture room, where desks, fixed to the floor, ascended in rows to the back of the room. If innovation was going to be applied then I could not think of a situation other than this, where the saying, 'old habits die hard', could be more apt. Success under these conditions should serve as encouragement for teachers bound to the most traditional of settings.

Composition of the 'class' of the Mathematics Open Day could not have been more heterogeneous, as it consisted, as has been noted, of young people of all the races in the Republic, virtually, representing Standards from 6 to 10 of the various different departments of education in the Cape.

### 3.3.2 NUMBER OF LESSONS

Because of a Spring Walk organised by the Simon's Town High School for Thursday - Friday was a public holiday - I taught a lesson a day to each of the classes for Monday and Tuesday, and wound up with a double lesson to each on the Wednesday. I had planned teaching each class quite differently, but it did not work that way, and the two classes were subject to much the same approach, with the exception that one received a stipulation about what to look for and investigate after viewing the film, while the others were left free to explore any avenue they felt the film had pointed them to. The Mathematics Open Day lesson was one lesson of just over one hour.

### 3.4 OUTLINE OF PROCEDURE

#### 3.4.1 STD 6A

The Shapes' film was the central feature about which the lessons revolved. The distinction between the two classes in approach was that investigation of the 'movement' in the film was asked for of the 6B class, while no such stipulation was made to the 6A's. The same manipulative materials were made available to each class, namely, geoboards and elastic bands, cardboard or thickish paper and scissors, dotted paper, quad paper and newsprint and koki pens.

After an introduction of myself and Sue, I described - a little too lengthily, I thought afterwards - what I was intending to do. The teacher, anxious about our reception by the class, had hastily informed us before the class entered that they had asked her whether it was an examination she was abandoning them to! I was at pains to allay their fears and this partly explains the undue time spent on making soothing noises to them.

Without more fuss, Sue showed the first stage of the film just once. I asked anyone to tell me the story of what was seen. Silence. I looked around. Pupils looked at one another, fidgeted, shifted about uneasily, then looked at me. The silence was significant. They had heard this preliminary patter about different lessons and so on. How much did it mean? They had to be convinced by more than talk. And they were waiting for me to make the first move.



To emphasise that what they saw was of importance, I was careful not to pass judgment on their observations, referring any pupil's comment back to the class, with a question like, "Do you agree?", and leaving it to the class's verdict. This is a tremendous break from the traditional teacher-vetting of pupil responses, as I was only too aware when I had to withhold the customary nods of approval, "yes's", "no's", "good's" and so on. I suspect part of the difficulty has to do with teacher's switching himself or herself out of the position of 'expert' into the less spectacular role of neutral 'chairman', more interested in keeping debate going than airing views.

I felt I was getting somewhere when some pupil-to-pupil discussion caught on occasionally, and this despite the formal rows into which the desks were arranged, in a room intended more for lecture/demonstration purposes than for class lessons.

Owing to class matters needing attention beforehand, lessons had started late and there was just time enough for a second viewing of the film, followed by more discussion when views were confirmed, revised or rejected. The class were keen to see the film again, measuring up what they saw with what had been said. However, activities had to be left over to the lesson on that next day, the class leaving the geoboards and other materials untouched.

### 3.4.2 STD 6B

With the experience of the 6A lesson fresh, activities started sooner, as introductions were cut short. Two showings of the film as before, with discussion after each, along similar lines to that in the previous class, were followed by group activities round the manipulative materials. Groups were asked to use the materials to reproduce what they had seen, looking particularly at movement in the film. This was to mark the distinction in approach between the two classes; the 6A's had not been asked to look at any aspect of the film in particular.

All groups worked on geoboards initially. Discussion and activity were lively and it was reluctantly that groups stopped and directed attention to my invitation to some group to demonstrate to the class what they had found. Quinton, a confident little boy, took the lead and showed the class on the overhead projector, using a transparent geoboard, what his group had been doing. I was careful again to abstain from comment, looking rather to the class for some response. Breaking out of the old teacher stance and role was what I was finding difficult, because of habit mainly, but also because pupils were used to putting questions and making responses via Teacher, and in that way, then, tend to hold one to the traditional role. That is why this lesson with its breakaway from norms, putting one out on a limb, does invite innovative action; but then one must go the whole way and have seating, as far as possible, encouraging pupils to interact with one another, not only in group sessions but in class discussions.

### 3.4.3 OPEN DAY

The procedure at the Mathematics Open Day lesson followed that described for the lessons of the Simon's Town High School classes, fitting however in this case into one lesson of just over an hour's duration instead of four, that totalled just less than two and a half hours for each class, in the Simon's Town case. There was the viewing of the Shapes' film initially, the class discussion, interspersed when desired by subsequent viewings, followed by group activity, using the same manipulative materials, and winding up with report-backs to the class, by the groups who chose to.

### 3.5 THE USE OF VIDEO RECORDINGS

A video camera recorded each lesson's proceedings and the analysis of the video tapes was used to gauge the effects of the Shapes' Film lessons.

Charles Hull, of the University of East Anglia, avers that while a "video-taped image of an event is more the event than is any person's recollection of it", it is "an arrest of the lived event" and as such can be used to reconstruct proceedings (1985, p.6). As he has demonstrated in exercises like the one referred to in the book, 'A Room Full of Children Thinking', "recordings of 'performances' are a rich medium for the development of analytic and reflective critique" (1986, p.92).

In a talk to students of the School of Education, University of Cape Town, in 1986, Hull described video-tape as 'information', and as a 'data source', and stated, "(Video-tape) seems the absolute ideal in terms of qualitative research", adding, "It's so rich", and, explaining, "It leads to reflection and consequently a construction of experience (1986-02-18). Hull saw 'data collection' as more a form of 'data creation', maintaining that through video, "we are presented with a new reality about ourselves", and he suggested when viewing the tape for analysis, "we play through a number of times and each time pinpointing something else" (1986-02-25). There was also, he pointed out, always the opportunity to confirm one's impressions by a replay of the tape.

## CHAPTER FOUR

### THE FILM AT WORK

#### 4.1 TEACHER'S ROLE

The change in approach that humanistic mathematics education demands of the teacher is nothing less than tremendous. While it is a traumatic experience for the teacher, the video-tapes fail to reflect the lack of confidence that accompanies the absence of traditional classroom procedure, and the pangs of doubt that assail one as power and responsibility are handed over to the pupils. The many occasions when one, out of habit, reverts to a usurpation of power, are some indication that Teacher is exerting considerable effort to keep to the new approach. However, it is quite easy, with reference to the tapes, to demonstrate the extent to which the children are thrown by the change of approach. They witness, as it were, Teacher unwittingly divesting himself or herself of the traditional role of expert they automatically accorded him, stranger though he may be. In fact, they feel forced into being party to it. One feels like a charlatan, persuading the children to connive with one at some malpractice designed to undermine established school practice. It is when it dawns on them - only by dint of sticking to one's guns - that power and responsibility have been accorded them and the processes they employ in confirming and verifying all that is said in the classroom, that they acquiesce in the take-over, and progress - as we shall see, is ensured.

#### 4.1.1 GETTING UNDER WAY

##### 4.1.1.1 INITIATING CLASS DISCUSSIONS

When I nudged the class along with requests like, "Won't you tell us what you mean by ...?", "Could you make it a bit clearer?", to even just, "Won't you repeat that?", I was placing on them gently, but fairly and squarely, the functions of listening to one another, with attendant disciplines of enquiring, discriminating and assessing - skills presumed essential for doing mathematics. However, while pupils - thrown in at the deep end as they were in these lessons, and more so at the Open Day lesson - do catch on surprisingly quickly to what is expected of them, it is for them, as for teachers, a departure from the traditional role they observe. They have to be convinced that Teacher not only means business but is capable of breaking with tradition. If this hesitancy of theirs was obvious in their reaction to me - a stranger - at the Simon's Town High and Open Day lessons, at the outset, how much more guarded, I expect, would they be towards teachers known only to them in the context of traditional teaching patterns.

My behaviour after the first showing of the film was crucial in all three classes. Children enter the class expecting the usual. Even from a stranger they expect the norms of basic schooling to be observed - Teacher the centre of it all, the expert holding all the answers and responsible for all that goes on, including their own learning. That there was no answer I was holding, that there was no one correct piece of information I was waiting for them to give was what I needed

to impress upon the Std 6A class in the silence after the first showing. I had to impress upon them that what they had seen was of importance, and was of importance to them more so, actually, than to me. My reception of the first tentative but courageous offering was being watched by all. To my question about what had been seen the word 'figures' had been put forward. Now it was all back at me. On my part, no nod of agreement even! I put it firmly back to them. "Does anyone agree?", I asked. The next response was again back to them for approval. It took a little while of this to persuade them that their statements - even quite 'way-off' ones in their eyes - were not being judged 'correct' or 'incorrect' by me. Gradually they began airing opinions more confidently. Practice makes perfect, and I found that one develops techniques that communicate to children one's new approach quite quickly. There were, for example, techniques of emphasising the listening attitude, on my part, and of insisting on silence from the class when one member was speaking, in order to accord importance to what each member says. My responses in the 6B class, afterwards, were more successful in helping pupils to say what they had seen. "What struck you?" seemed effective in evoking a response, while invitations to speak, like, "What do you think about that?", or "Anyone like to comment on that?", not only indicated assumptions on my part that they were listening, but acted as cues to draw them in to form their own opinions.

Preserving the right of the pupils to choose the line of investigation they take, while at the same time allowing the film to exert its influence in steering children along the

most productive paths for their learning, demands restraint from the teacher. It is so easy to say too much, drawing attention from the film's work. From the first viewing, the teacher's task is to emphasise the film, having the class recalling the images they saw, and imperceptibly being drawn to consider the main concept it is dealing with. Making a competition of remembering and being able to draw the 'first four pictures' they saw, or some such activity, is still giving reign to the powers of children, allowing them to come up with what they have seen, with the assurance that attention to its pictures leaves the film to point the children into the desired investigations. Elastics and geoboards provide about the nearest physical representation of the film's images, so that discussion proceeding on to demonstration with geoboards seems the natural next step and helps ensure that the film's influence is maintained.

The role of the teacher, then, is the crucial factor in humanising mathematics education. The initial stages when teacher and class face each other, after the first viewing of the film has been completed, are where new roles are established and pupils have their first taste of the new approach. The massive transaction taking place during these early stages can be appreciated from the recording of the lessons as they get under way. Vital in all this is the teacher's enthusiasm and determination, born out of the conviction that what he or she is about is of the utmost importance. The relevant transcripts are of the beginnings of lessons.



## STANDARD 6A CLASS

I start with the very first lesson, where the Std 6A's and I are left surveying each other after Sue has shown the film for the first time. Any vestige of confidence I had derived from the support of the film, associated equipment and so on, fast faded with the dying light of the film's last image. I was left standing there, no textbook to find refuge in and no structured lesson proceedings to refer to.

V1.0781 "And so? Can anyone tell me something?"

I move round to face the class and become the focus of their attention.

V1.0791 I try to smile. We look at each other: I, standing, expectant, they sitting behind their desks - almost crouching, some of them - watchful ...

The usual classroom lesson props like textbooks, pens and writing books are not there. ("There's nothing to distract us!") At this point the lesson's proceedings hinge purely on what we say and do to each other.

V1.0797 "What did you see?"

Eyes are on me. Silence. Anthony, the boy at the end of the second row, looks up carefully; he looks slightly uncomfortable.

"Shapes."

This is a mere murmur. Someone - Karel, I learn later, in the front row, second from left - is the intrepid one.

V1.0801 "Anyone?"

I've ignored him for the moment. Then I revert to the usual teacher noises, and address the class.

"Put your hand up, please."

And I proceed with more prattle about the importance of giving names, and so on, until it all dwindles to an incoherent mumble. Bravely I essay out again.

V1.0807 "Someone said 'shapes': who can add to that?"

A boy (Anthony), sitting with the trio in the third row, evinces discomfort as he looks round at the class.

"Who can add to that?",

I implore, with a hint of desperation in my voice. Silence.

Eyes are averted now; they're not on me anymore.

V1.0813 Anthony slowly lowers his face onto his hands on the desk. He sits on the right end of the threesome. The still, blonde boy in the middle looks straight ahead; he has been doing so virtually all the time. The boy on the right keeps shifting, looking towards Anthony; they have been exchanging sheepish glances frequently. Many are looking about now, turning to one another. The little boy in the second row, on the extreme right of the screen, chews at a thick pen or something, giving inadvertent glances in my direction between looking around at the others.

Are they as embarrassed as they appear? They do look upset. Are they really as uncomfortable as their looking about, their nervous movements and the sheepish smiles seem to suggest?

V1.0816 A trio of girls, right in the front, draw my attention. They've seen Anthony looking around; they exchange puzzled looks and smile covertly at one another.

V1.0827 "Do you agree?"

I asked the blonde one, Karen, in the middle in the front.

She nods shyly, then shakes her head, smiles and puts her hand in front of her mouth as if to stop herself laughing.

Pupils shift about in their seats; many are fidgeting.

Karen's puzzled look sums up, in a way, the various gestures and looks of bewilderment coming from all quarters of the class. I detect genuine discomfort on the part of the class; their looks of embarrassment, I sense, are for me. They look about at one another out of sympathy for me, asking in a way, "Who is this guy who seems so ill-equipped, possessing no structure or technique in the accepted sense, and who seems to be looking to us to get the lesson together?" My many years' enslavement to a traditional form of mathematics teaching means I am not at all surprised to detect an attitude that seems to say, "After all, when teachers ask questions, do they really put so much store on what pupils' answers are to be? They're only rhetorical questions, anyway, employed just to get the lesson under way, to pave the way for what teachers are to tell the pupils that day, a sort of tacit, token acknowledgment that the pupils are there, before the flow of knowledge - all from Teacher, of course - begins." And they are waiting for this to start, for the little 'introduction bit' to end, because ("Doesn't he know?") no one keeps it going for so long.

I don't believe they thought I was genuine about my expressed desire to hear what they had seen, until I come back on the same tack ("But this guy is persistent!") and I notice a marked change when I repeat the request:

V1.0635 "Come on, someone tell me: what was the story of the film?"

V1.0636 All heads turn my way. Movement ceases. All look at me.  
Silence. Karen turns to the pupil next to her; she looks  
thoughtful as she whispers something. She begins to raise her  
hand.

V1.0643 I miss it but notice a hand going up on the extreme left, at the  
front.

"Yes?"

Everyone looks to the corner, where Abri sits, hand raised. One  
senses a huge sigh of relief from the class because something -  
anything - is about to happen, as three little words are heard,  
"They're all quadrilaterals."

All eyes look back at me. I ask if anyone agrees with Abri.  
Many nod in agreement. They're still now, waiting ...

V1.0866 Hardly a twitch from me. Without giving any indication of my  
feelings about the statement, I ask,  
"Anyone else like to add something?"

Karen - still thoughtful - is saying something to her friend next  
to her, and

V1.0880 the boy at the far end of the Anthony-trio is mouthing something  
to Anthony. From what emerges later, (V1.0740), he could very  
likely be saying 'parallelograms'.

There is quite a long interval of silence, and a few look as if they are  
about to become disconcerted again. Some shuffle about in their seats.

I persist by asking them if they're sure they're all quadrilaterals.

Karen, engrossed, uses her finger on the desk to indicate some shape to her friend on the right. The boy, still chewing and without looking at him, whispers, tersely - and audibly - from behind clutching hand and pencil, to his friend,

V1.0880 "Say it."

His friend's hand goes up.

"Yes, Steven?"

The mood has changed; suddenly a certain composure about their movements is evident. People look thoughtful as they talk to each other. Steven had obviously spoken to his companion - 'Chewer' - in order to elicit the prompt that he received, Anthony and one of his companions in the 'parallelogram' incident must certainly have been consulting with each other, and Karen and her two were saying something quietly together. They seemed to be coming out of their shells.

The disbelief about my sincerity in asking their opinions has - with the persistence of my requests - given way to an earnestness, seen first in the whispered communications amongst themselves, and then in the readiness to contribute to the class discussion, which really takes off at this point. Is it that they're at last persuaded of my seriousness about their opinions? It's quite a lot to expect of them really. The traditional teacher-as-expert assumption has prevailed for a long time. It's actually a reversal of roles that has been under negotiation. Teacher is not in possession of everything, doling out to the pupils. He's really asking them, and he keeps on asking, actually treating pupils' comments as if they're of consequence.

I invite Steven to speak.

V1.0890 "Everyone has different sizes."

"Do you agree?", I ask the class.

Silence. Pupils look at me thoughtfully. Everyone looks serious.

V1.0906 "Steven, say it again, because I want to see if everyone agrees."

He repeats his statement. A girl behind Steven contributes something to what he has said. From a corner someone says, "No". I think it's Marius. He(?) adds something about their all being equal. At the far end, another girl is saying something across to Steven. Karen, looking very thoughtful, tentatively raises a hand. But people are looking attentively in Marius' direction.

"Yes, Marius?"

V1.0925 "There were a whole lot (of figures) that were the same."

Interest has captured the class. They're taking to heart my enquiries as to their agreement with one another and they're voicing their opinions. Power to assess contributions has been vested in them, they have realised it and they are starting to use it. It is as if the brakes have been released; a resistance to the change - an inertia, in effect, entrenched by years of non-participation, of 'listening' and of very little thinking - has gone. The class casts off its shackles, new life takes possession of the pupils and the initiative is out of my hands, virtually. This, despite all the constraints of fixed benches in rows in the classroom.

It's almost as if the 'new wine in old wineskins' has been disproved here: rather, I think, the virtually unused 'wineskins' of the powers and interests of children have been just waiting to receive the freedom due to them. It is more a case of the 'fields-standing-white-for-harvest' situation.

#### STANDARD 6B AND OPEN DAY CLASSES

It is interesting to compare the introduction of this approach to mathematics teaching in the other two classes. The Std 6B class have just seen the Shapes' film :

V1.1852 "What struck you?"

Darius, in the front,

"They were all squares and parallelograms."

V1.1857 "Did you hear what Darius said? Did you hear at the back?  
Anyone agree with what Darius said?"

Says one lone voice softly,

V1.1868 "Yes."

V1.1870 "Anyone got anything to add?"

There are the same periods of silence between the sparse lines of dialogue as was the case in the Std 6A class. However, pupils do not seem as disturbed; there is a composure about them. Perhaps I appear more confident.

Mario's hand goes up.

"Yes?"

V1.1873 "Squares and parallelograms with different lengths and shapes."

I to Darius,

"Does it tie up with what you were saying?"

He nods.

"Anyone else?"

A girl,

V1.1883 "They're all different squares."

V1.1886 "What do you think about that? Can you hear? Do you agree?"

Quite a few people nod.

Their acquiescence seems to be done without thought. Are they agreeing because they assume my call for responses is simply my method of checking that they're listening? My experience with the 6A's kept me persevering on the same tack.

V1.1905 "Anything else? Anything important that you saw? If you shut your eyes, I'm sure you can still see some of the images."

V1.1908 One girl shuts her eyes.

V1.1912 "... different shapes and different angles in a block."

V1.1924 "Who agrees with that? Who agrees with Robert?"

One girl has her hand to her mouth; there is very little movement and pupils, in general, are looking serious. While all are looking in my direction and seem to be attending, only four or five raise their hands in agreement. Silence.

Nothing more forthcoming.

V1.1937 "Would you like to see it again?"

A few indicate their willingness; the rest sit still, and then shift as if in preparation for viewing.



V1.1943 I come in with a little talk, starting,  
"What you see is very interesting to me." ...

V1.2037 The second viewing is characterised by the number of groups huddled together, engaged in earnest simultaneous viewing and discussion. Instead of discussion afterwards, groups are invited to use the manipulative materials. Most opt for geoboards and spend the next ten minutes concentrating on reproductions of the images they have seen.

Topics of discussion were their choice, but one sensed that all were working on whatever was right for them. Again I had been putting great store on asking pupils if they agreed, and again inertia discouraged or delayed responses, so that only persistence on my part - despite regrettable lapses like the reinstatement of authority of V1.1943 - prevented the whole process from foundering. This time it was not so much embarrassment as slight surprise and an automatic acquiescence, superseded - once the 6B's realised my concern was genuine - by a critical consideration of subsequent remarks. The phenomenon was more marked in the 6A class than the 6B's, where a greater variety of remarks were offered, once participation was accepted as the main business of the lesson.

Recourse to the video tapes of the third lesson, during the Open Day at the University, underlines the points discussed about the role of the teacher in getting the humanistic mathematics method under way. The class on that day consisted of pupils from different schools, who had arrived from normal lessons, bent on looking at experiments and other presentations; they were

to be entertained and experiences that made demands on them were not everyone's cup of tea. They could remove themselves if they wished, and this some of them did. From the opening invitation, "O.K. Anyone like to say?" (V2.2267), there was a similar sustained effort on the part of the teacher, persistent exhortations to them to comment, with frequent uses of "Do-you-agree?'s, and so on :

V2.2277 "We're seeing a square but if you cut it down ..."

"Could you hear at the back?"

Catherine continues, V2.2295. Then,

V2.2293 "That's Cath. Anyone like to say something more?"

"Anyone agree with that? Anyone?"

Silence.

"Something else?"

Pause.

"What else did you see?"

A voice is heard. I come in :

V2.2315 "Yes, someone ...?"

Silence.

V2.2318 "Please say it!"

"Come on, someone to tell us what they saw."

A general buzz. I pick up a word.

V2.2328 "Yes, let me hear that."

A voice,

V2.2329 "Parms."

"Let's hear that word too! I heard you say a word."

Something is said.

V2.2332 "Did you hear that?"

"Say it a little louder, please."

The Voice,

V2.2334 "Parms getting narrower and narrower."

V2.2335 "O.K. Anyone? Anyone like to say something more?"

"Anyone know that word? Anyone else heard that word before?"

"Someone like to say something more?"

And so it goes ... The reluctance in this case to speak should be seen in the context of the occasion. Open Day is ostensibly a day for looking and seeing - a day when displays are set up for one to view - not for participation. There was obviously a reserve about being drawn in. This, of course, besides the unfamiliar behaviour of the lecturer, teacher, instructor, or what have you, asking everything of you and contributing nothing himself - not even to add a yes or no to someone's contribution when it is forthcoming!

The process was under way. After the second viewing of the film, which followed immediately after the foregoing exchange, teacher exhortation changed inversely with the increase in class contribution. Viewing was followed by much freer discussion going by the sound of the general buzz.

V2.2460 "We've had two people say something. Anyone else? Come on!"

A voice from a group of four boys in the front,

V2.2462 "Each parm has the same area as each of those small squares."

"Can you hear?"

A general, "Yeah".

"Anyone not hear?"

"Anyone agree with that?"

From a few quarters,

"Yeah, yes."

V2.2474 "People agree?"

"Anything else?"

V2.2490 "Someone mentioned triangles just now; now we're hearing about  
parms. What's the difference?"

Says a girl in the front row,

V2.2498 "Two triangles equal a parm."

Another voice,

V2.2501 "Yes, yes."

The class was livening up. One has to keep in mind that a room occupied by young people of such heterogeneity, congregating like this for the first time - and, surely, for the last - hardly affords a venue conducive to free discussion, so that the situation was not comparable to that found in most classrooms. Once geoboards had been distributed conversation in the groups was more lively and bore fruit in the form of quite useful demonstrations in the Report-back session.

I refer briefly to the tapes of the last lessons when pupils were afforded viewings of themselves throughout the preceding lessons and were specifically asked about the classroom procedure in those lessons. Significant for the teacher's role was the following :

V2.0615 When asked why they are reluctant to contribute to discussion,  
even when asked to by the teacher,

V2.0616 "We are scared we talk nonsense," says one.

Asked whether they agreed, the rest of the class assented.

V2.0642 "And who are you afraid will laugh at you?"

"Pupils."

V2.0643 "Does the teacher ever laugh at you?"

Class, in unison :

"No."

Two traditional practices which teachers - especially old-hands - find extremely difficult to dispense with are, firstly, those of indicating approval or disapproval by "yes's", "no's", and various other noises, gestures or facial expressions, and, secondly, their abhorrence of incorrect responses, that has them assiduously smothering 'alien' information as rapidly as it rears its head. Acknowledging the strong prevalence of the latter attitude held by those in 'classical learning', Mary Boole (Tahta, 1972, p.16) cautions,

"But in science there are, there can be, no absolutely right impressions; our minds are not big enough to grasp any natural as a whole; everything depends upon drawing right conclusions from combinations of impressions, each of which is in itself inadequate and partially misleading; ...".

Hence Boole recommends that pupils be trained in the scientific method by being exposed to a variety of impressions, correct and incorrect.

Besides the inhibiting factors already attributed to them, the preoccupation of teachers with 'correct' impressions and their jealous retention of their role as sole arbitrator over the 'correctness' of every remark of the pupils are the two main culprits in pupils' non-participation in lessons. There was no laughter whatsoever at comments of pupils, in any of the transcripts quoted above. Why not, when impressions of all

sorts were encouraged? I conclude that it is when pupils work under the constrictive conditions imposed by the presence of an 'expert', holding the key to the 'correct' answer, that they feel nervous about being 'incorrect' or 'making fools' of themselves and then tend to laugh at one another, more out of embarrassment than anything else.

I was taking seriously all that they said; encouraging their responses and listening to them was not incidental and was not an act on my part while I waited for the big moment to squash them all with the 'punch-line', as it were. But they had to be convinced of this; hence my inordinate effort to impress them with my sincerity about listening to what they were saying and putting store on it, and by my insistence that others would be hearing what each one said. My determination - and of course my sincerity - does get through to them. When, after repeated calls to them to say something still has the 6A class sitting in embarrassed silence, I repeat :

V1.0842 "Come on, someone tell me. What was the story of the film?"

And Abri says,

V1.0850 "They're all quadrilaterals.

Steven, a moment later, adds,

V1.0890 "They are all of different sizes."

To emphasise I ask him to repeat it, and the highly significant - as I regard it - 'No' from Marius is evoked. It's of great significance because someone else besides the teacher has taken it upon himself to comment authoritatively concerning the correctness of someone else's response. My vesting them with the power has been accepted, appreciated and utilised. Subsequently, a second showing of the video has the class intent to a man.

Likewise initial overtures to the 6B class where I offer them rights to express opinions about the truth of statements wins their acceptance. The second viewing of the film is enacted in deadly seriousness. The huddle of earnest viewers is striking (V1.2037). As the discussion preceding the viewing demonstrates, it is not to prove or disprove opinions that I have aired that the class has recourse to the film with such intent; all relevant comment and statement on the first viewing had been theirs, not mine.

Abrogating the old role of 'expert' for that of 'chairman', who assigns to the pupils responsibility for their learning, might appear as a relinquishing of necessary status for maintaining effective schooling: Gattegno regards it as "subordination of teaching to learning" (1971), and consequently as deserving of the highest acclaim, and, states Shuller - a teacher engaged in similar lessons round film - "the rewards of working in a way in which teaching is subordinated to learning are great" (1983, p.41).

#### 4.1.1.2 PREPARATIONS FOR GROUP WORK

##### 4.1.1.2.1 PREPARATION OF CLASSROOM

More because I was unfamiliar about the classrooms I was to use and the pupils I was teaching, my organisation for seating and so on for grouping, was minimal. The classroom, with desks fastened to the floor in rows, was uncondusive to groupings of more than three pupils. There were advantages in that switching from class activities to group ones was easy, so that viewing could take place at any time without fuss and children could simultaneously watch the film, projected at the front of the class, and in their groups discuss and work.

##### 4.1.1.2.2 PREPARATION OF PUPILS

Short of routine announcements about their working in groups, as in the following transcripts, actual preparation was effected by inference. The only specific reference to group work made to the Std 6's was :

V1.0880 "As you're sitting now is satisfactory."

V1.0885 "I'm keen to have you working in groups when we do work, think and discuss."

After their second viewing of the film, as I introduce the manipulative materials I add :

V1.1544 "You're going to work in groups just as you're sitting now."



In the 6B class the class discussion and the introduction of the manipulative materials precede, and actually prepare the way for, the first reference to the group work, when I say :

V1.2000 "And you can work in groups."

The I casually make some token gestures for grouping.

"Darius, would you two work together, and Marius, you with the guy behind you, please?"

But more important than actual physical classroom preparation was the motivating of the pupils for group work. The sole direct references to groups were the statements quoted above, the main preparation being done very effectively by suggestion. The whole class discussion revolving round what they have seen serves to emphasise to pupils the importance of their contributions.

What skills does the teacher need, so that motivation arising out of the class discussions impels pupils to want to do work that leads to learning, that attracts them to activities of investigating and discovering? Obviously what is discussed must supply the ideas which give impetus to the learning process, and which constitute the material on which groups expend their efforts.

In order to have pupils feeling committed about what they say, it is important that the teacher makes much of their

statements, putting great store on the question of any and every contribution. As preceding video-tape transcripts show, the teacher has to make this his or her first priority, taking pains to establish, by meticulous insistence, to the point of fanaticism, the attitude of listening on the part of pupils with scrupulous attention to whatever is being said in the class. Getting pupils to talk and listen requires dedication and determination - an attitude that finds expression in the use of a variety of verbal implements.

Invitations to the different classes look like the following:

- V1.0807 "Who can expand on that?" to the 6A class,  
V1.0945 "Yes, something more?", to the 6B's,  
and to the Open Day class,  
V2.2267 "O.K. Anyone like to say? "Anyone?",  
and  
V2.2294 "Something else?"  
Silence.  
V2.2298 "What did you see? What did you see?",  
and addressing the class a little later,  
V2.2348 "I heard you say a word ... Anyone know that  
word?",  
and, after the second viewing,  
V2.2455 "We've had two people saying something. Anyone  
else? Come on!"

One feels like an auctioneer trying to solicit bids for an article no one wants! The whole point is, however, the accumulating of responses from the class. Every time a pupil contributes a statement, there is a commitment, an

involvement. One is, so to speak, obtaining self-appointed champions of causes. Every time one refrains from passing judgment on their responses, one vests, in the pupils, the responsibility for the maintenance of the accuracy and standards of their causes, enlisting, in this way, recruits for subsequent checking and confirming, and - ideally - for proving. Concurrent with appeals for contributions from the pupils come persistent enquiries after the state of their hearing. This exaggerated attention to the state of their auditory apparatus smacks very closely, I should imagine, of the solicitous approach of the ear specialist when attending to a patient. One's zeal, however, is amply rewarded when the unprecedented changes in attitude to learning takes hold of one's pupils. Old injurious habits are being uprooted; hence the note of determination so evident in the following statements :

Still with the class at the Open Day, in reaction to a pupil's comment, I ask,

V2.2280 "Could you hear at the back?"

A chorus of no's.

V2.2281 "Oh, would you say it, um ..."

"Cathy."

"... Catherine, would you say it again? Would you perhaps just turn slightly so that people can hear?"

V2.2298 "Anyone agree with that? Anyone?"

A little later, in response to a pupil,

V2.2463 "Can you hear? Anyone not hear?"

V2.2465 "Would you say it again, please?"

The teacher's preoccupation with their hearing what others in the class say, transmits clear messages about a transfer of responsibility. What each one says now is being heard and is being considered; not Teacher's statements only henceforth but pupils' statements carry import. It is their responsibility to assess and accept, and this imbues those making statements with the urge - as the following incidents show - to convince others. Emphasising what pupils say in order to implicate them in demonstrations, is what the teacher is about during the class discussion with the 6A's, as reflected in the following transcript :

Karen, the middle girl in the front row, has said something,

V1.0959 "Karen, repeat what you said."

She does so.

V1.0970 Anthony, the boy across the aisle behind Karen's group, shakes his head in disagreement with what she says.

A need has been created for Karen, and anyone agreeing with her, to demonstrate in some way the truth of what she says. Another need for confirmation - demonstration of some sort - arises when Marius commits himself with the following statement, a little later.

V1.1111 "They all have the same width - the same base."

Reacting to dissent from quite a few in the class,  
he insists, first of all to his partner :

V1.1114 "It is so!"

And swinging round to the class, repeats to all :

V1.1116 "It is so!"

Message received loud and clear! At any rate, if by no one else, by Marius! He has made the statements that everyone heard him do. Working to convince the rest of the class is his next most desired step. The stage is set! He is out to convince - not the teacher - but his own class members. He is bent on proving what he says, to his class, and Marius subsequently devotes his energies during group activity with his partner, Abri, to devising a proof that does convince them. In the demonstration, a resolute, confident Marius presents a proof of the statement he made with such rigour that, short of no one showing disagreement with it, he leaves a class of pupils visibly impressed by a quite superb show of logic and demonstration!

This did not occur before I had taken the opportunity of stirring up enthusiasm a little more. The first lesson with the 6A's finished off with the class discussion, leaving no time that day for group work. The next day's lesson with the class started with my recalling some of the statements pupils had made the day before; I mentioned the 'equal area' Karen had talked about, asking her,

V1.2642 "Do you believe it's true?"

"Yes."

V1.2644 I to Marius: "What do you think?"

V1.2645 "Not true."

Conflict! Marius had conceded earlier, in reaction to Steven's statement that all the quadrilaterals were different, that some areas were the same. Conditions are pregnant for striving to prove - for some demonstrating ...

As if there's not enough motivation, Teacher, unable to contain himself, plugs it, exploiting the situation even more:

V1.2647 "It will be interesting ... What is your name? ..."

V1.2648 "Karen."

V1.2649 "... Karen, if you ... Is there anyone who believes with Karen that the areas remain the same?"

Strong motivation for confirmation and for vindication is present. Does anything come of it? Let us examine just a few more possible contributory factors in relation to other groups before looking at the results.

There is a re-showing of the film, that has everyone interested. Even the two front restless ones look engrossed. Karen's trio are paying very careful attention. Nicoleen and the pupil on the other side are advising Karen as she copies diagrams from the screen.

V1.2690 I can be seen in discussion with Abri and Marius, while they watch.

I have not mentioned Marius' statement of yesterday to the class, but when the viewing ends, I goad the two of them by saying to the class :

V1.2733 "I said to Abri that they should go over it. They talk of heights and bases; they should follow it up and see what they can get out of it."

In a way I am publicly committing them - quite gently, I feel, and certainly with their approval - to some work that should demonstrate to us all what Marius has been saying. The transition to group work as a natural follow-on of the stand they have made with their statements and the video record of the concluding Report-back sessions - the fruits of the group work - is nothing short of spectacular.

#### 4.1.1.3 RUNNING GROUP WORK

##### STANDARD 6A

During the play-back of the video-tape of their class, the Std 6A's were asked :

V2.0751 "Some of you wanted to contribute but didn't. Why not?"

And after various comments :

V2.0798 "We're scared we'll make a mistake and other pupils will laugh at us."

V2.0826 "Have any of you an idea how a person can sit in a class and contribute without feeling shy?"

V2.0836 "If we sit together with our friends we're not shy to talk."

There was general assent when I asked the class if they agreed.

#### STANDARD 6B

When the Std 6B's were asked which they preferred between working as individuals or working in groups, the class agreed that groups was their first choice. Their opinion in general about group teaching is expressed in the following transcript :

V2.2075 "We give more attention than in the usual class," said Darius.

The comment from one concerning 'ordinary' mathematics classes was, "People just talk in that class."

V2.2082 "But you were talking in these mathematics classes too."

V2.2083 "Yes, but we were talking about the maths we were doing."

V2.2106 "Better than the normal maths," adds another, "because we can get somewhere and talk about it; other maths classes we just sit and listen."

The foregoing statements sum up the feelings of the two classes about working in groups, and provide also a fair guide as to the nature of the teacher's role. He or she is not to dominate the discussion of the groups, but rather to encourage the children to talk. Teacher, then, becomes a listener, questioning sometimes but more for the sake of



prodding in some direction if pupils are asking for it. The teacher's main performance has been a behind-the-scenes one. How one handled class discussions counts here because statements made during class discussions were crucial in determining what groups did afterwards.

Group work followed class discussion, allowing groups to discuss, and with the help of manipulative materials to express in concrete form what they had seen on film. The Std 6B's had group work on each of the three days, the Std 6A's on just two days, owing to routine class matters that interrupted their normal time table, and the Open Day class had just one session.

Group work is accepted as an integral part of lesson activity from the start. In fact, all other activities centre round group activity; both viewing of the film and class discussion form the basis of group work, film providing the content for discussion and investigation, and class discussion serving to start pupils thinking along the lines they are to pursue.

The 6B class started their group work early. Straight after the second viewing of the Shapes' video, during the first lesson, the class were offered geoboards and asked to think about the activities they were to pursue, in pairs or groups of three, and which they were to present to the class should they wish to. This was virtually an introduction to the

more substantial group session of the following day. The third day's lesson started with a viewing of the video, after which the class went into groups.

Was telling them about a Report-back where they would be required to demonstrate advisable? Did it inhibit group activity? It marks the start of pupils drawing together in groups, just because they feel the need of appearing together in a presentation. The 6A's are already pooling ideas during viewing, set in motion by my invitation :

V1.3707 "You may talk while you view."

The front right group has Anthony intent but not the other two, while Karen's group watch and discuss earnestly. Blonde Tanya and her group at the back are looking and then discussing alternately with a group in front of them, Tanya finally resorting to drawing.

The lights come on.

V1.2759 "Right Class, talk now about what you are going to work on."

I suggest activities for some, citing bases and height as a subject for Abri and company, movement round the border for Tanya and her group, reminding them really of what they have been thinking about up to now.

There is no time lost for Karen's group, who start immediately; Tanya's group are studying, with their neighbouring group in front, the drawings Tanya had made.

Teacher now has to withdraw as a 'public figure' and encourage the groups that look as if they need to be motivated. This was the case in the 6B class, where the Robin-Peter-Wendy trio started very reluctantly but, with geoboards and some attention from me, Peter and Robin began to show interest and Wendy was eventually drawn in.

Once groups have collected their manipulative materials the teacher assumes a different role - no more at the head or the front of the class, literally or figuratively. There is virtually no 'class' but many groups. The children are in a 'group mode', so to speak, not acting as individuals, so that one talks to 'groups' if one addresses the class. The video-tape shows me attempting the impossible: talking to the 6A class, who are already in groups, as if they are still a class of individuals and no one appears to be listening to me. Teacher's duties then are no more those of one dealing with a class; the function now is to join groups and facilitate the work going on in them. While I am out of the limelight in this role it yet remains as vital as it could ever be.

## STANDARD 6A

The 6A class has just started working in groups. I sit and listen to Tanya and her group while she explains and describes to the group. A girl from another group approaches and listens to Karen and stays to join her group just to watch and listen for a while. I leave as quietly as I joined the group to listen to Steven and his partner, who draw me into their conversation. The third day, immediately after the viewing, groups are at work and I am afforded ample opportunity for moving about, joining groups and listening - responding only when a group that I have joined asks me something.

During the viewing - which one should also regard as a group activity - I am in discussion with Steven and his partner. At the start of the group activity I am with a group of Anthony, Ronnie and Nico and another, and then - listening merely - with Tanya and her group. Then I move back again to Anthony and Company in the front. Next, I am seen again with Steven, who seems to be more working himself without the support of his group members. A moment later Steven calls me to show me something and one of his partners does show interest. Group activity for the 6A's ends then and the fruits of their labours and the effects of my work as facilitator will be evident in the demonstrations of groups during the Report-back.

STANDARD 6B

Group activity with the 6B's saw my role conducted in much the same way. There was a group at the back of the class - Robin, Peter and a girl, Wendy - who had displayed little interest during the class discussion, not participating in any obvious way. I had been standing at the back, and joined this group at the start of group activities, hoping by my presence to indicate interest in the group and so to help get them started on some film-related activity. I moved to a group of girls and suggested :

V1.2365 "Try and see if you can see something that stays the same. I'd be interested. You might like to show the class."

It is an attempt to motivate activity, presenting them some goal to work for.

The 6B class did have a short Report-back as a sort of introduction or rehearsal for the final one, at the end of the three days' lessons.

V1.2428 "I want somebody to come and tell the class what you saw."

Quinton, who was working with Robert and another, at the back of the classroom but on the other side of the Robin-Peter-Wendy trio, volunteered to come to the front and proceeded to demonstrate what he and his group had been

doing. This interim Report-back did have the effect of stimulating interest. After Quinton's contribution some discussion ensued, involving a number of pupils.

Subsequent group activities afforded me more opportunities to visit groups, when again I was able to talk to the Robin-Peter-Wendy trio. I mention particularly visits to this group because of subsequent events during the Report-back. The activities of the 6B groups were punctuated occasionally by a reviewing of the video, requested by a group but involving invariably all the groups. During such events group activity and viewing were often carried on simultaneously.

The Open Day class had their group activity with at least three teachers in attendance at certain groups - teachers who attended the lesson and just stayed sitting in with the group of pupils in their neighbourhood. I visited a group and listened; Sue - in the role of a teacher - showed a group how to press out shapes in paper on the geoboards. As with the other classes the pupil-to-pupil discussion and the group-generated activity were important; teacher's role of 'expert' addressing the class or overtly conducting all proceedings was conspicuous by its absence.

#### 4.1.1.4 CONDUCTING THE REPORT-BACK

A Report-back session, in the form of group demonstrations, was treated as a voluntary affair, groups being asked to 'show' the class what they had found, working on the assumption that they would want to tell others their findings. It seemed to work in all the classes concerned. I found it important not to appear anxious where a class like the 6B, initially, seemed shy to start :

V1.2484 "Now I want you to tell ... I want someone to come and tell the class ... to demonstrate what you felt you saw."

And again,

V1.2495 "I leave it to you, hey. Which group's going to start?"

And Robert and Quinton come to the front.

I found myself allaying pupil anxiety about the hidden 'correct answer' when my first invitation to a group in the Open Day class to come up to prove produced the reaction,

V2.3367 "To prove what?"

Where pupils have been discovering and learning from one another (and occasionally from the teacher) during group work, now what a group has come upon is about to be presented to the class and so the teacher's role as expert must not intrude at all. The role resembles that effected during the class discussions, when one 'chaired' to a certain extent, acting almost like a compere.

Sometimes persuasion, like that in Steven's case in the 6A class, was without effect, and in the case of Anthony in the 6B class. In both cases I refrained from forcing the issue, preferring to let pupils see that it truly was a voluntary participation that was involved. Rasheed and Company in the Open Day class were persuaded, my exhortation,

V2.3440 "Rasheed, why don't you go?"

being just that extra prompting to what his group had already been targets of, from the group behind.

Routine queries about pupils being able to hear are the order of the day for the teacher, assuming importance when they extend to insisting that a demonstrator, as with Helen, leave her seat and conduct her explanation from the front.

I took care to refrain from comment after the end of the first pupil demonstrations, asking simply for questions or further comment from the class, so that when Rasheed, for example, talks to the class at the Open Day session rather than to me, it is particularly thrilling in that pupils are addressing pupils. The teacher has to discretely deflect the habitual talking-to-teacher that pupils fall into so easily, as at the start of Paula's talk 'to the class', as this is destined to destroy all hopes of pupil interchange getting off the ground. Once given the lead, pupils were happy to talk to one another, and Cathy in the Open Day class, when I responded to Rasheed's demonstration, needed no second prompting :



V2.3510 "Any comments, any queries?"

V2.3511 "Yeah, what about what's ...?"

V2.3512 "Ask Rasheed."

Whereupon Cathy turns to Rasheed

V2.3513 "Oh, what happens when you ...?"

The teacher acting as 'chairman' in the Report-back sessions is assuming a role that is unfamiliar but exciting in the exercise of a power - not to control pupils - but to bring them out of themselves, to see them expressing themselves freely and confidently. The ultimate in pupil-to-pupil interchange occurred when difficulties were experienced during a group's (or individual's) presentation. Because, as in the case with Cathy's demonstration during the Open Day lesson, I kept quiet when there was a call for help (V2.3611), help arrived from a different quarter (V2.3615). Rasheed came to her rescue. Pupils were helping pupils with no interference from me! And they solve their difficulty themselves and commend themselves, when Cathy exclaims, to accompanying applause,

V2.3650 "Oh, well done!"

There was a similar call for help - with the same exciting developments - in the 6A class demonstrations. With Teacher acting the mere onlooker Marius comes to the group's aid.

## 4.2 IMPACT

### 4.2.1 ON CHILDREN

Consideration of the effects that the granting of autonomy has on learning affords the teacher who planned and initiated, who attended and experienced with all concerned the process of its traumatic birth, great joy. It resembles that experienced at the view of a dry field's responses to gentle soaking rains that descend on it. The dormant field of Learning revives as the long-awaited growth of the many life-skills related to mathematics awake in children, stretch themselves and become active in phenomenal growth. In one's recourse to the tapes one is afforded 'before and after' pictures that testify quite conclusively to the power of the impact of the lessons on children.

From stances of neutrality and states of nonchalance one witnesses, as was the case with Robin, Peter and Wendy in the 6B class, the withdrawn pupils, who are present in the mathematics class in body only, actually according full attention, involving themselves in discussion and enthusiastic group activity. Such developments lead eventually to active participation in a Report-back demonstration to the members of their class. There were in the same class pupils like Samantha or the unknown girl on her own, who, conspicuous for contributing nothing at the start, became active participants during the Report-back. One is reminded of Borenson's 'exciting process' (1986, p.36) when in the 6A class the exciting transition of individual

lifeless spectators into active learners bursting with life takes place before one (V1.0925).

Transference from the state of a couldn't-care-less dependence on Teacher as supreme controller to one of alert responsibility occurs when faced suddenly with all the choices, when what you say contributes to the learning. Such was the case during the Open Day lesson, when Paul and his group were in trouble in the demonstration and the finding a way out was their prerogative and responsibility (V2.2713). The child becomes a 'problem poser/chooser' in the 'personal construction of knowledge' (Gordon, 1978, p.263).

#### 4.2.1.1 FREEDOM TO INVESTIGATE

The freedom to investigate involves, firstly, the freedom to choose one's topic of investigation. The Shapes' video lent itself to a freedom of imagery, that short, infrequent viewings of it triggers off, so that, in Std 6A for instance, Tanya was afforded the opportunity to consider the movement of the shapes, while in the Std 6B class, Quinton chose to look at mirror images in opposite corners, and Tracey had this to say about the video,

V1.4400 "I found that it was like a square in the beginning and it got bigger in the middle and it went smaller when it got back to the side."

Then Samantha, for the first time, contributed :

V1.4405 "Here it was all like big and then it got stretched and then it came back to this side and ... like squares ... like three squares and they stretched."

The freedom of choice in the use of manipulative materials was accorded the groups at the sight of their activities and during them, as in the Open Day lesson, when after introducing cardboard and scissors amongst others, I added,

V2.2438 "There's also paper if you want to cut out something you were saying and show us."

Freedom was a characteristic of pupils' activities in their groups. The group in the row second-from-front is a case in point, when a boy, while watching the video, pulls the geoboard from his partner and proceeds to work at it with the help of the girl at his side (V2.3222). The girl in the middle then uses it with him (V2.3229), the former one intervenes again (V2.3235), so that all are seen involved round one geoboard. Next, the boy takes over (V2.3235 to 3240), and he continues operating it while the middle girl suggests something and the end girl adds to it (V2.3251). And so the discussion develops while the board changes hands as each requires it in order to try out or confirm some issue (V2.3285).

Where microphones were placed at groups' tables one can gauge the extent to which the free use of materials influenced the discussion in those groups. Belinda in the 6B class had not up to yet displayed signs of having any confidence. The dialogue that follows shows Belinda instructing a group partner - quite correctly - as they count squares.

- V1.3669 "Yes, you don't count from where you end off; you count from where you start again."
- V1.3670 "One, two, ... No, I dunno ..."
- V1.3671 "You don't count ... That's where the block starts ... that block ... that point there. That's where the block ends."

#### 4.2.1.2 FREEDOM OF LEARNING

Is the freedom that pertains in the class discussions and the group activities, as demonstrated in the foregoing section, meaningful in terms of learning? One might tend to regard the Report-back sessions as the main platforms for the demonstration of the learning that was achieved.

Examination of the video-tapes of Class Discussions and Group Activities, however, reveals that these activities also serve as records of development in learning. The activity of learning assumes a corporate dimension; it becomes richer as pupils learn from one another and together develop skills and construct a body of knowledge.

What does study of the video tapes impress upon one about the effects granting of freedom in the different activities achieves? There is a freedom about the learning. Pupils engage in corporate learning activities and development occurs in a variety of interactions, as can be seen when analysing the following key learning issues :

(a) Learning to work together

The freedom from shyness and fear of sharing with others is worth consideration. Working in groups apparently frees pupils of these inhibitions. I asked both 6A and 6B classes a series of questions during the evaluation of the lessons :

"Some wanted to contribute and didn't. Why?"

and

"Is there a way of asking to help you to contribute?"

The first question evoked the reply, "We're scared we'll make a mistake",

and

"Scared we talk nonsense ... people laugh at us."

The second question produced the following response :

"If we sit together with our friends we're not shy to talk."

The 6A class were seeing themselves working in groups on the video tapes. We pause ...

V1.0094 "Anyone to comment? Your opinions on how you sit?"

"It's better to sit with friends."

"Why? You like sitting with friends?"

"Yes."

"For learning purposes?"

"Yes."

While viewing themselves at work, discussion with the 6B class on the same topic went as follows :

V2.2613 "Any comment about the way you're working there?"

"Working with groups."

"One can argue with others."

"Arguments destructive or constructive?"

"Constructive."

"Do you do this normally in a class?"

"No ... work by oneself."

"Which do you prefer?"

"Working together."

Viewing of the film seemed to have pupils' attention; one appreciates, however, how a competitiveness engendered by asking who could reproduce the first, say, four images of the film, could raise levels of observation. This is something all can do. In turn it breaks the ice, making it easier for pupils to contribute in class discussions or in their groups. Observing, after all, is what one is after, initially, because then the film can do its work.

One detects the forging of a class spirit - a group feeling that is disposed to work in a positive co-operative way. The response of a Std 6B pupil, "We learnt to work together", to my request for comments about the use of the lessons they had just had, was significant. The collaborative nature of the class activities was crucial in its development. The shared

viewing of the Shapes' film and the ensuing class discussion, where opinions were exchanged, commented on and accepted or dismissed by themselves, impressed upon them that they together were engaged in a learning venture, that the responsibility was theirs. When asked a question about the intensity with which they viewed the Shapes' film, their sense of collaborative nature of the process is revealed in statements like "We wanted to try to see if it was like that", and "'We're trying to confirm your first impression". The response, "Yes, but we were talking about the film", was an effort to underline the difference of that talking from their own reports about talking in the usual mathematics classes. Relate this statement to another pupil's saying that the lessons in question were structured for them to talk, comparing 'other maths classes' where they 'just sit and listen', and it is evident that they approve of 'talking about maths'. The tenor of these statements expresses the corporate sense of the class appreciating the responsibility invested in them, and which they had assumed gradually but inexorably over the four lessons - 'gradually' only because I had found the 'handing over' of power so arduous and so unnatural and had done it so stingily.

(b) Learning to observe together

First showings of the film catch pupils' interest, because of the associations films have for children; but shared viewing is a factor that contributes to attentiveness. When, after pupils have aired opinions



about what was seen the first time, a second showing is suggested, attention is intense because now they can confirm or refute what was said.

"You may talk", my remark to the 6A class before their first viewing of the video in the second lesson may sound insignificant: it is nothing of the sort. It helps ensure that a free attitude prevails during viewing of the Shapes' video tape. Karen, who is trying to prove areas constant, watches intently, discussing at the same time with the group and copying diagrams from the screen. For the 6B's, a freedom about the viewing of the video in the second day also prevails. I cannot see anyone who is not looking at the video except Peter, from the row at the back, and even he sits up and takes note. His was the 'quiet' group referred to: it is rewarding to watch them now. Robin - without taking his eyes from the screen - points out something to Peter. Wendy is looking at the video, making some comment to herself, it seems. Now Peter points out something to Robin. And these were the three whom I had labelled 'non-starters' as far as any mathematics was concerned!

A subsequent viewing by the same class shows Helen and the rest of her trio and Darien and his partner all engrossed, as if it is their business to observe - their skill that is to point out to them what to see. What compels them? Class regulations? Teacher's pervasive discipline? I asked them when we were all watching the video tape of the lessons :

V2.1613 "What made you watch so?"

V2.1620 "Trying to confirm your first ... um ... what you ... um ... first thought you'd seen."

What did the freedom achieve?

I asked the 6B class :

V2.2066 "Does something else come to mind that you felt you learnt?"

V2.2068 "We learnt to observe."

(c) Learning to discuss

One detects a refining process going on as pupils use terms and develop on one another's language. Starting in the initial discussion in the 6A class with a general comment about 'shapes', stepping up to 'quadrilaterals' next, then an irrelevant statement about 'parms' which they discard, to a key comment about their all being figures of 'different sizes'. The last comment draws a 'no' from Marius who counters that there are a 'whole lot that were the same'. With that Karen adds :

V1.0933 "All the figures could fit into one square, exactly into one square."

Marius improves upon it with :

V1.0938 The same area!"

Karen refines it to :

V1.0948 "All figures' areas are the same."

The freedom of language usage places the responsibility on pupils to search for better terms, as we see Marius later on correcting someone in the following two incidents :

V1.1092 "They're all the same width."

V1.1097 "They're all the same base."

V1.4467 "It's important that the volume is the same."

V1.4468 "Area."

As language becomes more accurate so concepts are defined more precisely and understanding grows. A dialogue between two pupils in the 6B class discussion illustrates the point. A girl is contending what Anthony has just said :

V1.3186 "He says the sides are equal but they're not; they're parallel."

V1.3187 They're parallel and they're equal."

In the same class there is the case when Lindi, in her search for the correct term is helped by another to say 'angles'.

One has no record of the actual statements but the scenes of the intense discussions round the viewings of the Shapes' video make speculation about possible knowledge acquisition amongst the viewers, in the interchange of ideas, very exciting. The viewing done in the 6A class on the second day are a case in point,

as also are the earnest discussions around the two 6B viewings on the second and third days.

(d) Learning from one another

This presumes the art of listening. Pupils had to be challenged to listen to one another. There was firstly the listening in class discussions and in groups. Because the teacher abstained from commenting there was a freedom experienced during the class discussions that could implicate the pupils quite seriously in the exercising of critical awareness, especially because the teacher was not vetting any or every response as to 'correctness'. The responsibilities of listening to one another and assessing contributions were being offered to them. But was I sure that all the 'Can-you-hear's and 'Do-you-agree's were being heeded and were achieving their aim?

Class discussion in the Std 6B during the first session left me unconvinced about the extent of their listening to one another until Quinton, replying to a remark from one of the class, said :

V1.2569 "It's the same as Baker said when ..."

I was further persuaded when in the same class, during a second class discussion, Tracey's observation drew the response from Darius that is recorded below :

V1.3197 "I think that both of the top sides equal to the bottom ones."

And Darius,

V1.3198 "That's what Anthony just said."

In the 6A class, Marius' classic 'no' to Steven's conclusion that all quadrilaterals were of different sizes, left me in no doubt about the quality of the listening that was operative there.

A second issue was the listening in the Report-back sessions. Demonstrations that groups were free to present in the Report-back sessions were the climax of the foregoing learning experiences. How effective the freedoms of thinking, choosing, discussing and participating had been should be seen in the contributions of work of the groups. What does the fact that the Robin-Peter-Wendy group features in the 6B class Report-back underline? Demonstrating with a geoboard, first Peter then Robin (Wendy remains in her seat), explain :

"... mainly consists of parms and squares; that's what you can see."

"Each square when the elastics get formed, they form more parms all the way round."

Is there provision for differentiation? Do the wide range of abilities with regard to language, background knowledge and the varying degrees of self-confidence gain expression? The opportunity to express what one

sees according to one's own insight is free for everyone to use. In the 6A Report-back, intellectually sophisticated demonstrations about areas from advanced pupils like Marius and his partner, Abri and Karen and her group, occur, while Tanya contributes on the less scholarly topic of the movement of the shapes about the border of the figure. Even during the initial class discussion more able and less able pupils could both express their thoughts without hindering each other. In the 6A class, Tanya could concentrate on movement :

V2.1190 "The figures appear on the edge of the figure. They were in a circle: they aren't in the middle."

Karen, in the same session concentrating on area, could sum up the whole point of the Shapes' video with her succinct remark about the areas all being the same, without causing the lesson to end there and then. Rasheed, of the Open Day class, also could sum up after the second showing of the Shapes' video, with the following statement, and yet not put an end to all discussion :

V2.2423 "Each parm has the same area as each of the small squares."

How is it that Karen's and Rasheed's statements did not terminate all subsequent mental activity and simply close off all proceedings? My non-committal attitude, my responses to such revealing comments from the more mentally quick pupils leaves the decisions to the.

pupils. My replies to Rasheed's statement leave all the passing of judgment to the class :

V2.2430 "Anything else?"

V2.2435 "Who saw something else?"

Because the issue is left open more is expected and so the discussion resumes with the additional comment by a pupil of another group :

V2.2501 "The two squares that they use ... the two triangles on either side equal the parm."

Consequently, pupils of the ilk of Karen and Rasheed are not left bored having said it all, for while others battle to understand what they were on about, the onus on such advanced pupils is to devise demonstrations that convince the class of their conclusions, because no 'expert' has taken it upon himself/herself to pronounce them correct. And so the lesson proceeds with everyone in the class challenged according to his or her capacity.

(e) Learning to choose

As the film concerns itself with area, investigations arising out of viewing the film that revolve round this concept, or allied ones, should achieve the most in learning experience for pupils. How to avoid pupils getting on to other less productive issues while preserving their right to choose is important. It is

so easy for the teacher to intrude in work that the film does very effectively by itself. I found it better to refrain from making comment when the film was shown and during class discussion immediately after. (When I did stipulate 'movement' as a factor, in the case of the Std 6B's, odd irrelevant issues emerged.)

Gattegno (1981, p.21) warns about seeing the content of his film as "knowledge to be acquired by all". Years of learning on the premise of one 'correct' answer that has to be found 'one' way has stultified children's doing mathematics and even smothered the process altogether in the majority of cases. Significantly it was more obvious in older children, the groups of Std 8 and 9 pupils at the Open Day lessons being hesitant about their investigations in case they were not on the right track'. Group activities were ending off; I had invited groups to demonstrate or prove what they had discovered, whereupon I was asked, "To prove what". What anxiety did this not reveal?

"The showdown's coming; the answer's going to come out."

"All this time we've been working, we've either been right or wrong."



#### 4.3 TEACHER'S ROLE

Pupils feel interested when their opinions are taken into account, especially when done so by the other pupils. This is where teacher expends great effort to refrain from comment when pupils are talking, leaving the pupils, rather, to assess the truth or whatever of their colleagues' statements. It takes much control, at first, on the part of teachers to respond to comments that pupils, out of habit, direct to them, with questions like "Do you agree?" or "Who agrees?". It did not take long, however, before pupils became used to being asked like this and began to recognise and to act upon this assumption of the teacher that they did have something of worth to contribute.

Preserving the right of the pupils to choose the line of investigation they take, while at the same time allowing the film to exert its influence in steering children along the most productive paths for their learning, demands restraint from the teacher. It is so easy to say too much, drawing attention from the film's work. From the first viewing, the teacher's task is to emphasise the film, having the class recalling the images they saw, and imperceptibly being drawn to consider the main concept it is dealing with. Making a competition of remembering and being able to draw the 'first four pictures' they saw, or some such activity, is still giving reign to the powers of children allowing them to come up with what they have seen, with the assurance that attention to its pictures leaves the film to 'nudge' children into the desired investigations. Elastics and geoboards provide about the nearest physical representation of the film's images, so that discussion proceeding on to demonstration with geoboards seems the natural next step and means that the film's influence is maintained.

#### 4.4 THE STEPS THAT FOLLOW

Whether all the problems of mathematics education can be solved by humanistic mathematics remains to be seen. Will the investigative approach illustrated here close the case of problematic mathematics teaching?

There are many imponderables that need serious consideration. What mathematics should be taught in the mathematics class and can what should be taught be taught in the way illustrated here? Dr Pirie states,

"It must be said at this juncture that investigations are not a universal panacea for all our mathematics teaching ills."

Certainly the very real learning benefits as described here, from this learning method, merit the attention and consideration of all concerned with mathematics education. More than that, they beg implementation in the classroom.

From there the many questions that arise can call us to research for the answers. That our appetites are whetted and enthusiasm engendered so that the necessary research steps can be taken is the crucial factor.

## CHAPTER FIVE

### REACHING THE TEACHER

#### 5.1 INTRODUCTION

"There is an irrefutable case against assessment only by time limited written examination papers", states David Kent, in an article, 'Alternative Forms', in the Times Educational Supplement, 1986-10-10, p.46. Writing on 'Investigations as Coursework' as envisaged in the GCSE, the new examination dispensation operative in the United Kingdom from 1987, Kent is aware however, that the "reasonable measure of objectivity and uniformity of standards" (Kent, 1986, p.46) that written examination papers afford make teachers reluctant to introduce other forms of assessment. The demands in terms of assessment objectives are heavy, including as they do : "oral response to questions; discussion of mathematical ideas; ...; practical work; mathematical investigations and work of an extended nature" (Thompson, 1986, p.41).

While the idea and terms related to humanistic mathematics have been in circulation in the United Kingdom in the recent past, it appears the necessary groundwork - the in-service and teachers' college courses - needed to introduce the practising teacher to the developments and to nurture the growth of teaching skills necessary to apply them, have been inadequate.

Ian Thompson, writing in an article, 'When Does the Real Training Start', in the Times Educational Supplement, 1986-10-10, p.42, offers the following advice :

"Open-ended investigational work certainly demands that teachers adopt a non-traditional teaching role, and if the inclusion of such work is not to become a last-minute appendage to the curriculum, teachers need to be offered in-service courses where they can develop some of the necessary skills ..."

One deduces that the introduction of the GCSE has caught teachers napping, so to speak, when in the same Times Educational Supplement, Dr Susan Pirie, in an article, 'Your Investigations', p.42, points out :

"The personal cost of an attempt to approach pupil-learning in a novel way must not be underestimated. An act of faith is involved in the resolution "I'll stop telling them everything and let them discover mathematics for themselves" - faith in pupils' inquisitiveness, faith in one's own class management, faith in the gospel that mathematics can be learned in this way."

#### 5.1.1 TEACHERS' NEEDS

Faith without some confidence about the power of the process to improve the learning of mathematics smacks more of wishful thinking and forms no basis for the large scale introduction and establishment of the humanistic mathematics method.

Teachers need to see the process in operation. As David Kent states,

"... the most important part of our (the teachers) learning experience must come in doing the work with pupils, meeting colleagues and establishing criteria upon which we make judgments and establishing a consensus of opinion" (1986, p.46).

More than "lists of 'tips and hints for teachers'", says Ian Thompson (1986, p.42), "are needed to bring about modifications in teachers' attitudes to mathematics and changes in teaching style if investigational work is to become an intrinsic part of mathematics teaching". He adds that resistance to change that implies "a big change in the teacher's role" (1986, p.42) is to be expected. I have found that even young students resist the idea of such changes. Fear of mathematics in general makes any proposed departures from the acknowledged but highly suspect style based on "exposition, consolidation and practice" (Pirie, 1986, p.42), undesirable in the extreme. Prospective teachers emerging with confidences shaken by the experience of twelve years of school mathematics are loth to leave the known way, no matter how detrimental it has proved itself to be - even in their own experiences.

While it is new and exciting for all, the heaviest demands in the assumption of new roles in the humanistic mathematics learning fall on the teacher. He or she has to demonstrate to children not only a sincerity about changing but convince them that the capability to effect change is there. I wonder if their confidence about such capability is not slightly less firm when it is placed in their very own teacher, known so well to them filling the traditional teacher's role, than, as with my case and with the school children of Simon's Town High and those in the Open Day session, when it was placed in a stranger. The teacher and his or her class embarking on a

totally new approach is a phenomenon smacking of the blind leading the blind. There could be the tendency - even the desire at times when at a loss for what to do - for both pupils and teacher to revert to customary practice.

All the more reason for care and forethought when approaching practising teachers about changes they might make, for lesson notes that help equip teachers to meet the new demands and that call on them for their co-operation in a collaborative research venture to produce effective procedures that other teachers can follow. Getting under way in the new style was for me especially difficult, and while a list of questions and remarks that encourage free expression and participation are useful, I found calling to mind what it was I was aiming for served as the sort of guiding star to help supply the right remark at the opportune time.

Miss Brink, the classes' teacher, was quick to remark on the positive attitude the 6A class had evinced in the third mathematics lesson of the day, one following on the double I had had that day, and which she had been anticipating very reluctantly. She told of being able to accomplish a lot with them and at the end, to her surprise, still found them amenable to her hesitant proposal regarding homework! This was the class she herself had described as 'switched off' as far as mathematics was concerned. Gattegno's statement with regard to the Nicolet-type films he was introducing comes to mind : "One can expect a substantial and even spectacular improvement

of attitude and of performance ..." (1981, p.4). It adds incentive to the idea of teachers using lessons like this once every fortnight, say, as boosters for flagging zeal, not to speak of other more specific benefits in children's understanding of mathematics. The school experience and the Open Day lesson showed classes gaining entry to degrees of activity and thought that is impressive, bearing in mind the usual attitude displayed in mathematics lessons. Again one recalls words of Gattegno, saying of his films that they afforded "opportunities to do as much as can be done for as small a cost as possible in time and effort" (1981, p.4).

Most teachers, just by length of practice, are more set in their ways than their pupils, and consequently do take time to surface, shedding with difficulty past practice and finding dispensing with crutches, like text books, unnerving. Placing pupils and their powers at the centre of consideration in planning the next move, made demands on one's discernment of pupils' needs that left one exhausted ... but excited. The unfamiliar role of arranging procedures in lessons according to pupils' responses and needs was risky and nerve-racking for me. Pupils were now my point of reference. After each class, plans for succeeding lessons were hastily reformulated, and all because the pupils were being taken into account, not the topic, not the next page of the text book, not anything else. I was looking at the pupils to know what next step to take; from their most recent responses came my guidance for future moves. For the first time, really, I was thinking about what

pupils say and do, actually gearing my lessons round them and planning the next steps that the lesson takes in terms of their perceived needs. I was exposing myself fully, I felt, for the first time to an intense experience of child-centred learning.

Granted the introduction of the GCSE in the United Kingdom is a help; while it has left many teachers feeling ill-equipped and inadequate and at a loss about how quite to proceed, there is a tremendous sense of release in that the shackles of the traditional prescriptive examination system have been removed.

Crucial to us, yoked still to a rigid examination system in South Africa, is the question of the purpose, energy and work that preceded and pioneered the inception of the GCSE in the United Kingdom. How much research into effective teaching of mathematics was done, how thorough was the dissemination of such research to practising teachers and how faithful and courageous were the classroom teachers who ventured to apply the new ideas in traditional settings where a variety of conditions were often quite inimical even to hints of change?

#### 5.1.2 TEACHERS AS RESEARCHERS

The necessity of a pioneering stage to developments in education is accepted by all concerned with growth. No one questions the following official statement of the position of the National Council of Teachers of Mathematics in the United States of America that

"Significant improvements in the teaching and learning of mathematics result from the systematic development of research-based knowledge" (1986).



In the same article, the NCTM continues :

"But it is the mathematics educators themselves, trained in the content and methods of teaching mathematics, who must assume leadership roles in conceptualizing, communicating and implementing research related to mathematics instruction."

Mrs Hilda Barry, South African Teachers' Association representative of the Mathematics (Primary) Study Committee, states in her report to the SATA Conference, held in Cape Town, June 1987,

"The Executive Committee of which I am a member, is now involved in investigating 'How Research results can best be made available to the teacher in the classroom.'"

In a sustained effort to make research not only available to teachers but to actually engage them in the research process themselves, Charles Hull, of the Centre for Applied Research in Education at the University of East Anglia, concludes that

"contrary to common expectation, group meetings which bring together teachers attempting classroom research for the first time and a support team from a university are not the most productive occasions for the development of a 'culture' of teacher research" (1986, p.104).

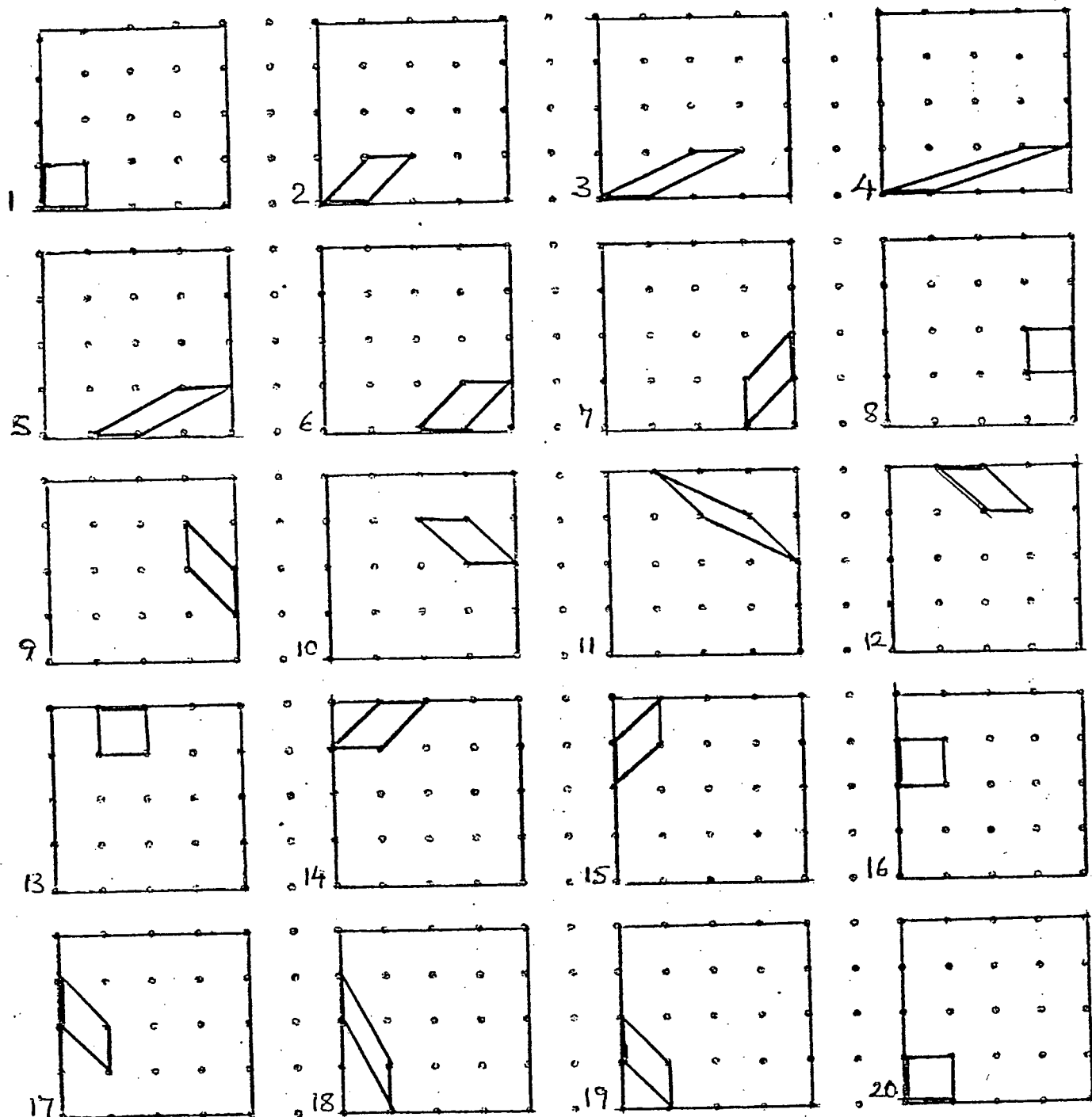
He found that 'one-to-one meetings' between the advisors or facilitators of the research and the teachers conducting the research were initially more valuable in "generating both confidence and independence of thought" (1986, p.104), and that small group meetings were only conducive to meaningful sharing once research in the different schools where the teachers were based had been the subject of discussion just between 'advisors' and 'operators'.

### 5.1.3 SHAPES' FILM LESSONS

Because of the history of resistance to change and the acknowledged difficulty of the application of research results in classrooms, strategy and effort need to be all the more skilfully and vigorously applied in the interests of the cause of improved mathematics learning. It is urgent that teachers experience the humanistic mathematics method of teaching. Teachers who see demonstrations of the Shapes' film and its accompanying lessons, as described in his thesis, can see for themselves impressive results in attitude and performance of the pupils concerned and can gauge the potential for effective learning of mathematics that the application of such an approach holds for their teaching. It is urgent that as many teachers as possible hear of the humanistic mathematics method, see it demonstrated in lessons based on investigational topics, try it out for themselves in their classrooms and then share their experiences with colleagues involved in the same research. For this purpose the formation of small research study groups of primary school teachers is necessary, keeping in mind the necessity for one-to-one encounters between teacher-in-research and the advisor.

APPENDIX A

SHAPES' FILM SCRIPT



## APPENDIX B

### ANALYSIS OF VIDEO-TAPES

First tape : V1  
Second tape : V2

#### C O N T E N T S

#### COUNTER NUMBER

SHAPES' FILM I	(Used in the lessons for this dissertation)	V1.0007 to V1.0098
Std 6A :	Lesson 1 Class Discussion and Viewing	V1.0370 to V1.1560
	Lesson 2 Class Discussion and Viewing Group Work and Viewing	V1.2770 to V1.3080 V1.3080 to V1.3360
	Lesson 3 Group Work and Viewing Report-back	V1.4925 to V1.5300 V1.5300 to V1.5750
	Lesson 4 Evaluation	V1.5900 to V2.1180
STD 6B :	Lesson 1 Class Discussion and Viewing Group Activity Report-back	V1.1590 to V1.2160 V1.2170 to 1540 V1.2570 to V1.2770
	Lesson 2 Class Discussion and Viewing Group Activity	V1.3370 to V1.3690 V1.3690 to V1.4020
	Lesson 3 Group Work and Viewing Report-back	V1.4050 to V1.4370 V1.4370 to V1.4700
	Lesson 4 Evaluation	V2.1180 to V2.2170
OPEN DAY CLASS :	Discussion and Viewing Group Work and Viewing Report-back	V2.2180 to V2.2530 V2.2530 to V2.3320 V2.3360 to 3830

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U N I V E R S I T Y   O F   C A P E   T O W N

F A C U L T Y   O F   E D U C A T I O N

C O U R S E   P A P E R S

M A S T E R   O F   E D U C A T I O N

T H O M A S   B R Y A N T   V I N C E N T

AUGUST 1986

## ON LEARNING MATHS : A "STUDENT'S" INSIGHTS

Two books were my source: Ledermann, W., 'Complex Numbers' and Markushevitch, A.I., 'Complex Numbers and Conformal Mappings'. Each with its different approach was to enlighten me sometimes and frustrate me others, as I used each text to introduce me to complex numbers, the purpose of the exercise being to gain insight into the learning of maths by actually placing myself in a learning situation.

Ledermann introduces the subject by briefly tracing the development of number systems from natural numbers through the more elaborate systems, as needs arose. Arithmetic, the science of numbers, is based, as he says on the fact that numbers can be added and multiplied. These two 'laws of composition' are subject to certain rules, and their existence and their 'mutual relation' will represent the common property of all numbers. These will lead us and we will constantly refer to them when new number systems have to be introduced (Ledermann, 1967, p.1). The set of rational numbers confers the power to apply the four rules of addition, subtraction, multiplication and division without restriction, 'always excepting division by zero' (Ledermann, 1967, p.2).

Markushevitch begs the question of complex numbers by showing previously counting number systems portrayed along a line, visually demonstrates their basic operations and their rules along this 'number' line, and then asks, virtually, 'what about the whole plane of paper on which this line runs - its two-dimensional setting?'. And then venturing out on the plane, with other lines - lines at



angles to our number line - we accord 'numbers' to these lines and proceed to see how they perform with the adding and multiplying operations. Only once established, having earned our respect by proprietary behaviour under the operations, are the identities of these numbers examined and we find strange things that they can represent, like  $\sqrt{-1}$ . Only then is it recognised as solving a long standing problem: the equation of the form

$$ax^2 + 1 = 0$$

March 27th

I borrowed three books on the subject from the library. The first one I glanced at was A.I. Markushevitch's 'Complex Numbers and Conformal Mappings'. Often I had seen vectors in association with complex numbers. Why? Markushevitch's book should answer this one.

I left the subject; obviously more pressing matters to think about.

May 27th

I found another description of real numbers: the set of all decimal fractions! Quite neat. That every real number can be represented by a vector on a line, was just a variation on the usual number line description, without the term 'vector'. But 'vector' does imply that there is more to come, whereas the number line seemed to end there. One does not think of a segment doing much else than lying along the X-axis, but one expects vectors to rotate possibly. The scene is set, so to speak, for developments on real numbers; saturation was not achieved when real numbers were introduced.

W. Ledermann's 'Complex Numbers' was picked up next. Complex numbers are mathematical objects for which addition, multiplication and other algebraical operations are defined subject to certain rules which closely resemble the corresponding rules for the more familiar types of number. With the introduction of  $i^2$  as  $-1$ , it was incumbent on mathematicians to see if complex numbers can be added and multiplied and if all seven fundamental laws hold.

May 28th

He traces the development of the sets of numbers up to real numbers, and then mentions how "one of the simplest algebraical questions remains in an unsatisfactory state when only real numbers are available" (Ledermann, 1967, p.4). Some quadratic equations would have no solution, unless

$$x^2 + 1 = 0$$

could be solved. So to find the square root of  $-1$ , the introduction of a new set of numbers is necessary. Hence the introduction of  $i$ , and complex numbers.

May 30th

I am into the algebra of complex numbers and getting a little lost. Why? I don't think the rigour of applying the properties of the basic operations impresses me as being so necessary. But I appreciate that one needs consistency. I wonder if it's the algebra - the finicky symbols - that makes it all seem pointless.

The pace being self-imposed, as it is, has the advantage that one has the freedom to speculate and question but the disadvantage that, with no pressure, one prognosticates and little headway is made. I suppose the secret is to have the pressure without there being a threat. Is this possible? I think going in one's own direction helps; choosing what avenue to investigate and knowing that the investigations will be regarded as significant does help in eliminating fear. But then when the steps are there in the textbook prescribing the path one takes, it seems a bit senseless to wander off along paths of one's own choosing. One feels the mathematician who created this maths has already been along other paths, found them dead-ends and rejected them. Who wants to waste time trying them again?

Margaret Donaldson, 'Children's Minds', maintains that children "have a relatively well-developed capacity for making sense of situations involving direct and immediate human interaction" (p.76).

She mentions "disembedded thought", with reference to maths that is removed from the realities of one's experience (Donaldson, p.76). The 'linguistics' of the algebra renders Ledermann's explanation into a form that is removed and remote for me and I lose interest.

June 3rd

Real numbers, I see in Ledermann's book, are special cases of complex numbers, where the imaginary part is zero. The operations of multiplication and addition are used to obtain certain interesting answers. I find it intriguing how, having created something in

maths, we investigate how the 'creature' performs. Ledermann describes the interesting result obtained when a complex number is multiplied by its conjugate (Ledermann, 1967, p.7).

For the following two days I work through the algebra of squaring complex numbers and other related operations without much hassle. Yet I do find myself getting put off by the abstract nature of the involved calculations. One is following the logic of each step but getting confused about direction and whereabouts. Interest wanes quickly when purpose eludes one.

I begin to yearn for work that is more comprehensible and visual and decide to go back to Markushevitch's book!

June 11th

I find this book (Markushevitch, 1962) is interesting. Less algebra and more graphical representation in the form of geometry of lines and angles features in this presentation.

Vectors on a line are related to the real numbers; vectors on a plane represent the complex numbers; and what of vectors in space? I'm interested to know and I'm surprised to find myself making enquiry about a topic in maths in this way.

How much more homely suddenly complex numbers appear to be! To think that imaginary numbers can be illustrated on paper visually! So real numbers lie on the X-axis, or parallel to it, while imaginary numbers lie on the Y-axis, and mixtures of the two occupy any other part of

the plane. This visual image is satisfying.

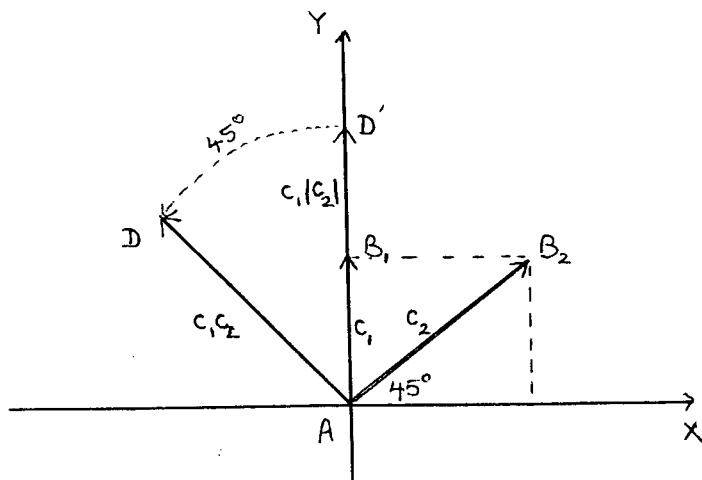
Note: we are looking at the representation or 'photograph' of complex numbers before we really know what they are. Markushevitch (p.4) points this out and reminds his readers that in life one often sees the portrait before the original (p.4). I compare the ease with which I've appreciated this treatment of complex numbers with Ledermann's abstract, remote algebraic presentation.

June 12th

As questions were applied to the vectors of real numbers, so we reserve the procedure for the vectors of complex numbers and find the commutative and associative laws holding just as in addition with real numbers. I'm not convinced it was all so necessary.

June 13th

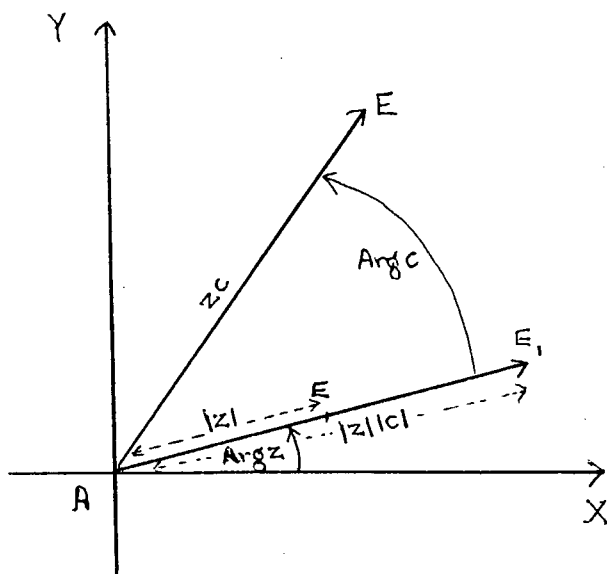
How much easier to understand the concept of absolute value when it is reproduced in a figure. I'm cautious about multiplication, however, the diagram being misleading for me; I confuse it with addition.



The illustration of the distributive law is satisfying - using similar triangles as it does. A question arises: If a triangle is rotated about any point, are all the sides rotated the same amount? And what of all the points on a side? Am I really interested in pursuing this? On my own? How would I feel as one of a class of thirty? They would all be getting on. I'd say to myself, "Stop messing about! Keep up!"

June 25th

Multiplication of a vector by a vector implies, in the general case, both extension and rotation of the original vector, pure rotation when the multiplier's absolute value is unity and pure extension when the multiplier is a real positive number (Markushevitch, 1962,p.19). I find this easy to accept when it is illustrated in the way Markushevitch does.



June 27th

The term 'conformal mapping' defines geometrical transformations in which angles between any two curves of the transformed figure are constant. Homothetic transformations, rotation and displacements are examples of conformal mappings.

What is all this for? Margaret Donaldson refers to our feeling most at home when immediate goals and intentions and recognisable patterns of events are present, and explains that there is "an incapacity when reasoning unsupported by human sense is called for" (Donaldson, p.80).

June 30th

Representing the earth's surface on a plane entails distortion, but it can be done so that "angles between various lines on the earth's surface remain unchanged" (Markushevitch, 1962, p.29). So many questions arise at this point that are not asked and, of course, not answered. I leave it for today.

July 1st

I put off so! Actually getting down to work on the maths content seems so hard for me. I'll do anything but get into the maths. I spend any amount of time looking for suitable books, finding the applicable titles in the books, arranging the books in order of attack, but find it so difficult to apply myself to the reading of the maths. Why? I'm afraid. I'm scared I'm going to meet something

I'm not going to understand. And where is the interest? Eliminated by fear, I should say. I feel I'm not up to what I might meet; there are going to be statements that I shall not be able to accept. There'll be no-one to answer the queries that arise, anyway. Perhaps there's a lurking suspicion that they are 'stupid' questions; no-one else would be interested in them. Does an attitude like this arise from working on one's own? Would being part of a group help - a group of people who accepted one? Surely they would listen to one. Would hearing others' queries not reassure me about my own - that they were not so off the mark? Of course, groups aren't the sole answer; a sympathetic teacher whom one trusted could serve the same purpose, providing an ear that one felt invited questions.

At this point I decide to start again with Markushevitch's book. A second time round and understanding has increased. His elegant approach is appreciated more this time.

Markushevitch also starts with the number line but defines immediately segments of the number line as vectors on a straight line - 'directed segments' (1962, p.1). I suppose the algebra of cartesian co-ordinates is also not restrictive but vectors, with the implicit emphasis on movement, do seem to give hints, begging questions and inserting the germ of expectancy, so that, to a certain degree, departure from the number line is anticipated.

After basic information about vectors is given, departure from the number line is further anticipated by the explanation of the argument of a vector, written as  $\text{Arg } x$ , for any real number  $x$ , the argument



being the angle between the vector representing this number and the positive direction of the number line.

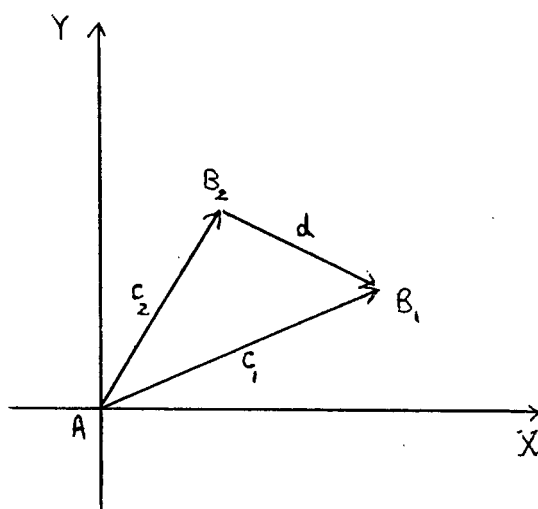
Question: Why can't  $x = 0$ ?

Answer : The number 0 is represented by a point, not by a vector. We're unable to say anything, according to Markushevitch, about its direction or about its angle with the number line, and this makes sense. Thus about the vector of zero length, no statement can be made concerning argument (1962, p.1).

Addition and multiplication are shown by figures using vectors to represent real numbers, of course just along the number line. Then lines at angles to the number line are introduced, quite naturally because the paper plane is there. Logically one appreciates - as Markushevitch explains - how real numbers in relation to the plane of the paper are a particular case of other numbers. Thence, marking out the plane with the two axes at right angles emphasises how the vectors on the X-axis (the number line) are a subset of all possible vectors. Complex numbers is the name given to the universe of vectors on the plane.

By preserving the geometrical form of the rules found for the addition and multiplication of real numbers, Markushevitch leads us to investigate addition and multiplication on the vectors of complex numbers. The commutative and associative laws are seen to hold. The rule for multiplying the complex numbers, other than real numbers, is preceded by the introduction of concepts of absolute value and argument for these numbers. (Now  $\text{Arg } x$  need not be confined to  $0^\circ$  and  $\pm 180^\circ$ .)

His insight in complex numbers and the cartesian plane enables Markushevitch to relate the two so thoroughly consistently that the presentation of the topic is a satisfying experience. Subtraction with vectors, for example, is seen to be the inverse of addition because  $c_1 - c_2$  is obtained by a joining operation of  $d$ , from end of  $c_2$  to the end of  $c_1$ .



Division is more intricate. I appreciate it only after some thought which doesn't appear in the statement, so one doubts one's capabilities. Only very capable, quick pupils comprehend fully what is presented in textbooks. 'Batting about', trying a number of solutions before hitting on the right one is considered untidy, irrelevant and not part of the solution.

The deduction of the vector representation of  $\sqrt{-1}$  wins one's agreement because the conclusions are seen to be based on the rules we started with, and it is satisfying because the use of logic in the individual is catered for. Starting, as many texts do, with  $i^2 = -1$  and  $\therefore i = \pm\sqrt{-1}$  violates one's own logic system, ignoring it in effect, with the

implication that it does not deserve any attention. Markushevitch's explanation, although it presented us with a polished system, did show respect for its readers' powers of logic. Doesn't any 'telling' in maths amount to violation, to a disrespect for children's powers, when it switches out the child's doubts?

## MATHS IN SCHOOL

There seems to be confusion about the purpose of teaching maths in schools. Is it for 'doing' maths or 'using' maths that we plan so and struggle so? We need to be clear about the two activities.

As I have been studying complex numbers, it dawned on me that I was intending to 'do' maths. I was aiming to understand the reasons that lay behind each statement in the text. I was trying to understand the answers - which is what the work represents - to the questions that were asked when the system was being built up. The difficulties encountered, as recorded, arose because the textbook does not show the questions that gave rise to its neat, complete set of answers.

I have not seen any textbook that does show the questions, the different attempts - some futile, some helpful - that led to the correct solutions in the creation of a maths topic. What happens, in fact, is that schools end up teaching their pupils how to 'use' maths, something they don't do very well because using maths in Geography, for example, is best taught by geographers, and so on.

Maths classes need to attend to teaching how to 'do' maths; teachers need to engage pupils in what is called mathematizing.

## FEAR

Awareness grows only out of seeking, asking and knocking. Thinking about my reluctance to even go to the book and read on about complex numbers, I compare it to a type of paralysis - a condition born of fear. Why fear? It's a fear, mentioned earlier, of asking questions and exposing myself. (I'm into deep things now!)

I go back to 1940. I remember early years at school, pre-primary years in a school in Pretoria. My father had come to join the S.A. Army. I was seven, and coming from Lourenco Marques had had no schooling up to then because the Portuguese believe in starting school late. My extreme timidity, I can remember, gave way to relief when teachers seemed to like me and school seemed relatively easy. Until mental arithmetic reared its ugly head! Instead of answering first as usual, I found myself left behind, stumped for answers. Numbers presented me with problems, when the response had to be given quickly.

In later years in Std 9 and 10, I remember maths lessons mainly because I was always asking questions, it seemed. And this habit earned for me ridicule - even anger - from both pupils and teacher. Despite this, I would persist in putting up my hand because things just would not 'click' for me. I seemed over-cautious about accepting any mathematical statement my teacher - a well-known personage locally, who had written the textbook we were using - made in the course of his inadequate explanations. I remember his remarking once: "Vincent, I think your trouble is you're too suspicious; you will not accept any statement in maths. You seem to feel that it would be dishonest if you take something that I tell you

as being true".

I suspect this caution stemmed from my 'hermit crab' mentality; an attitude of the utmost reluctance to venture anywhere out of my shell, never mind an actual move of shell and all! My readings this year, however, lead me to think that there could be more to it than just this negative disposition that I've had to contend with. Statements of most of the authors from which I have quoted tie up with that statement of my maths teacher.

My reluctance to ask questions - to even put myself in a position where questions could arise - originated, I guess, from the frustration I used to cause both teacher and fellow pupils by my persistence in asking questions. But why the need to ask questions continually? Marshall Gordon refers to a "radical conception of education" signifying "a sense of going back to the roots, the subjective roots of the mathematics experience" (1978, p.252). Was I - and am I still - in search of the experience, that Gordon goes on to state, constitutes part of the gaining of knowledge and which involves one in creating that knowledge and, which Gordon avers, is denied students of maths by the "finely polished, impersonal mathematics curriculum" with which they are presented (1978, p.252)?

#### MATHEMATIZING

Reflection on my persistent questioning during maths lessons at school, which my wrestling with this topic of 'new maths' had recalled, occasioned insight into maths teaching which, for me, has been very fulfilling.

I started by wondering why I had felt impelled to ask, firstly, and then why questioning in a maths class should have been disruptive. These two questions acted like two powerful facilitators, drawing together and reforming much of the material I had collected up to then, by ear, eye and whatever, during the M.Ed. course.

First, why the compulsion within me to ask? Does this imply a deeper level of integrity on my part? Are those quicker to accept than I was, less honest than me - satisfied with a more superficial acceptance than I? It took a conversation between my wife and an art lecturer from Stellenbosch University, during a week-end break, to supply the answer.

My wife, an art teacher herself, maintains that everyone can draw. That many people don't believe this and truly think that they cannot draw is because they don't look, she believes. What many prospective art students try to do - and do unsuccessfully - is to draw someone else's 'looking'. For example, they draw an eye from pictures that other people have seen and drawn, but not as they see one.

Consequently, in preliminary exercises she has students drawing with their left hands, with their drawings covered over so that they cannot see what they are drawing, putting down negative spaces and a variety of methods that compel looking and that play down the outcome, the product on paper.

The art lecturer at the University of Stellenbosch made the distinction between principles and formulae, in his conversation,

where formulae constitute the 'copies' that students make of what others have seen, while principles make up the drawing arising from one's own looking. He states that he is a protagonist for getting down to principles, starting his students on the basics of light and shade, and so on. Only after they have concerned themselves with these principles, he says, will they be able to produce works that constitute art.

Returning to maths, the process referred to as mathematizing comes to mind here. No, I don't credit myself with more integrity than my fellow pupils, because I insisted in asking more questions; I was, however, feeling the urge to mathematize. I don't say the others were not doing so; I am sure the very capable pupils were able to do this, but not because of any intention or effort on the part of the teacher to have them do so. Teaching in those days - and, tragically, today - did not make provision for mathematizing; in fact, it actively hampered it.

Mathematizing is doing maths, and that isn't what pupils are expected to do in school in the maths class; they are presented with maths and have to receive it. Mathematizing is asking questions. If I am to 'do' the maths as the mathematician who produced the topic that I am concerned with did it, I shall have to ask questions. Every point that the mathematician, in the act of creation, reached came only because he asked himself questions. If I am to understand a mathematician's product I shall have to ask the questions that he or she asked. I ask a question from a point that I am at and the answer to it I can integrate with what I already have. Learning maths is

like a 'screening': every new item of knowledge received has to be accepted and made part of what is already there, changing the original if need be in the absorption process. Caleb Gattegno puts it neatly: "Now, as soon as we shift from acquiring facts through memory to acquiring them through functioning, we unify our experience in the duration of one life - for we always build on and integrate with what already exists and do not simply pile one fragment of information upon another - and we recognise that inner meaning is more important than outside authority" (1971, p.13).

What is this method of maths teaching that has caused me to find myself threatened when investigating a new topic? It's important to ask, because it is this same method that is used today in almost every school everywhere. My experience of being discouraged to question in a maths lesson is not unique. More than 'discouraged', questioning was - and is, I believe - resented, because it disrupted the lesson. And when one thinks about how one teaches maths, it's quite natural that it should be. We teach by presenting and we expect pupils to receive. We are handing them something. One might - through a twinge of conscience - allow questions (some teachers go so far as to implore questions) but the whole act of giving something doesn't really invite questions. Questions are not part of the package; they are interruptions. They signify discrepancies in presentation or, worse, they reveal incomprehension on the part of the questioner. Questions, in short, denote something has gone wrong! And then we wonder why our pupils are reluctant to ask when we are presenting them with some topic! Yet the originator of a maths topic got into it by questions!



Presenting students with the 'polished' (Gordon, 1978, p.252) curriculum, that effectively cuts out all the wrestling and 'batting about' that went into its creation constitutes an offence to senses of logic and consistency of the students - their 'apparatus' for doing maths; it amounts to a denial of a right accorded everyone who has a mind that works.

Like the exercises that the art lecturer, referred to earlier, organises for his students, in order to have them doing art, pupils in maths have to problematise the material they learn. Investigations have to be set up in such a way that the element of fearlessness necessary to heed the prompting of one's curiosity, to pursue intuition and to allow questions to arise and to consider them, is encouraged. Fearlessness cannot be taught or given; only the conditions for its encouragement can be arranged for. Situations for experimenting and investigating can be set up and supportive influences provided - conditions characterising 'radical education', in Gordon's terms. Investigatory maths puts students (or pupils) in the right setting for meaningful development in doing maths.

The activities teachers organise for mathematizing vary according to purpose, like needs for precision, attention to detail, working to rules, detecting pattern and finding proper ties, making generalisations, or looking for a gap and being prepared to take it. Depending on which they prepare for, teachers are required either to conduct exhaustive searches into concepts of particular maths topics or to pay careful attention to creating the social conditions that

present maths as something amenable and within the reach of all, dispelling that removed, remote aspect that many people see about maths.

Harking back to the analogy with art, the works of art that grace galleries resemble works of maths in the sense that they also are finished products. They represent the best that artists can achieve, when at the pinnacle of performance. Of late, among the many art books illustrating the best of works, are books appearing that reveal the practice over years that went into the making of the works of art. Maths has only textbooks of finished products, produced by practiced mathematicians. This is acceptable if, like engineers, we are only interested in applying the end products. But pupils of maths are expected to work through, understand and make these perfect works their own, so to speak. A more historical approach where the search, the attempts that were unsuccessful as well as the successful are considered, would be useful in encouraging pupils to try themselves, and the tendency to modestly opt out from working oneself would be discouraged.

When, despite the hazards that textbooks and much of the teaching present, pupils do mathematize, it can be done at any level of maths at any time. I found myself switching into it when I was understanding and when I was not threatened. However, when I could not grasp an explanation, any further mathematizing ceased. An insecurity about building onto faulty knowledge like putting a brick over a hollow place stops one's freedom in thinking.

Maths teachers are notorious for being performers in their classes, delivering faultless solutions to problems with apparent ease and confidence, taking a leaf, no doubt, from textbooks' impeccable performances. Pupils are never aware either of the real use teachers do make of textbooks' explanations and examples in practice sessions before classroom appearances! We are seen asking no questions of ourselves about passage from one step to the next and yet we have the temerity to urge pupils to 'ask questions' when they are to go over what we have done.

This stems from pride partly, and also from an obligation we feel that, because of our qualifications in maths, it would be wrong to admit that we did not know everything about a topic we were to teach.

The effect that such behaviour has on pupils is to stifle their 'doing' maths. Before we can ask pupils to mathematize we have to be seen doing it. Here lies the dilemma of maths teachers who sincerely want to encourage their pupils to 'do' maths. Trying to start on an equal footing with one's pupils in a topic - which would constitute genuine mathematizing - either means we're engaging in bluff or that we are unqualified to teach the topic. A possible solution is that since mathematizing concerns itself with skills more than it does with factual knowledge, the topics used are, in effect, incidental.

## INVESTIGATIVE MATHS

The need in the maths class for giving pupils opportunities to mathematize has led to investigative maths, where situations for investigation are set up, that require no previous platform of

knowledge. Knowing now - more so than ever - the crippling effect that a gap in one's knowledge has on doing maths, I appreciate the freedom that this 'starting from scratch', as it were, gives a pupil.

Investigative maths answers the problem about fears of making mistakes or of taking the wrong path in seeking solutions. The instruction, 'Investigate', on being presented with some simple phenomenon related to numbers, carries no prescriptions about 'right' or 'wrong' paths one chooses.

Undertaking investigations lends itself to group activities and the reluctance I felt, sometimes, in pursuing some problem of complex numbers because of lack of enthusiasm, could be avoided where groups feature. Hearing questions from others stimulates the posing of one's own questions, and of course, enthusiasm is infectious.

Are we, then, to scrap syllabi, omitting all content which requires the use of memory as a factor when one 'does' maths? Because the skills of mathematizing are all-important in being able to 'do' maths, are trivial puzzles to replace the maths of schooling? That one incident I experienced (recorded under June 11th, p.5) where one was engaged in a topic of which one understood the steps although the fundamental idea had not been defined is significant. Mathematizing, as I have mentioned, is a 'mode' one switches into at any stage of maths; the skills involved in mathematizing are important, and once these have been made one's own, so to speak - established initially by investigative activities in trivial items - one is confident to apply them more widely.

Margaret Donaldson warns against the scrapping of maths that involves 'disembedded' thought just because it is difficult to do, and lowering standards, "abandoning the arduous task of functioning without the support of the world of familiar events" (1979, p.82), adding that the emphasis would have to be shifted without "denying the significance of intellectual skills" (1979, p.83).

Like the yeast, inserting just occasional mathematizing activities using investigations serves as a meaningful start. But how necessary it is that we start.

Dawning insights afford one deep fulfilment: they also challenge and disturb when they point to desperate needs for action. In the field of maths education the cry for change is very urgent.

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## TOWARDS BETTER TEACHING:

### DECIMALS AND PLACE VALUE

William Willson and Geoff Wain, two influential maths educationists visiting from the U.K. in early July, joined our M.Ed. class for a discussion. Mr Willson, with reference to decimals, said that with the advent of the pocket calculator, decimals were gaining in status in curricula while the importance of fractions, it appeared, was declining. There were moves afoot, according to Willson, for one or two curricula committees, on the strength of this, to keep fractions out of syllabi - with the exception of a few of the widely-used ones, like halves and quarters, and so on.

This view, representing as it does, extreme opinion at present, does, however, vindicate trends in thought about the relative importance of fractions in education. Research into children's understanding of decimals - even this year - indicates deficiencies in basic understanding of the topic, to a degree that, in the words of a researcher: "Most 15-year-olds do not understand decimals" (Foxman, 1985, p.19). Foxman's conclusion about 11-year-olds, a group on which he also conducted research, was that they "... have only a fragile grasp of place value" (1985, p.19).

Derek Foxman led a team called the Assessment of Performance Unit (APU) that spent five years testing 150 000 pupils aged 11 and 15 years in British schools, between 1978 and 1982 (Foxman, 1985, p.19). A verdict based on the results of the second maths assessment of the National Assessment of Educational Progress (NAEP), reviewed in



The Arithmetic Teacher, April 1981, involving decimals by 9- and 13-year-olds in schools in the U.S.A., demonstrates a comparable lack of understanding of the topic. (Carpenter, Corbitt, Kepner, Lindquist and Reys, 1981, p.34).

Recommendations of researchers are that teachers review their approach to decimals from the very introduction of the topic, and research that can contribute to effective teaching by suggesting new approaches and indicating changes in emphasis, will be very useful. That "the established tradition of university-based educational research" (Hull, 1985, p.86), is regarded as concerned with problems irrelevant to activities in classrooms is a common view. According to Hull, teachers complain that researchers exploit the children that they conduct research on for their own ends only and often use the data inconsiderately (1985, p.86). Further, teachers find the 'jargon' used in research as a hindrance, barring from access everyone except the individual engaged in extra studies. In reading research, one appreciates the ease with which one could succumb to subjectivity, often where it is not taken into account by the researcher who is intent on taking great pains to observe the strictest objectivity. One recalls words by Marshall Gordon (1978, p.252) that "truth follows from personal intention", rather than the diction that one follows truth wherever it may lead. If research leads one to conclusions foreign to what one holds strongly to be the case, strong resistance against a tendency to look for extenuating factors for the data is necessary. In one case I quote, it is the independent abstractor, feeling strongly about the merits of a certain teaching approach, who

furnishes explanations for rejecting the conclusions that run counter to his!

Then there is the question of demonstration recommended by Gattegno when he states: "Teachers may be impressed by academic research, but they are not influenced by it. They demand that ideas be translated into classroom action" (1970, p.50). Until this is the case, the "inattention" teachers accord traditional research "is their judgment" (Hull, 1984, p.11) and not indication of inability to accept change. In effect, Hull supports Gattegno when he adds: "One solution is to switch from attempts to disseminate 'packaged' innovation or research 'results' to dissemination of the research process itself" (1984, p.12). But Hull is critical of both Gattegno's idea of demonstration when the teachers are to be demonstrated to, as he is of teacher educators and academics designing research methods to impose on teachers (1984, p.12).

If I am not to miss the point of this project, presenting numerous descriptions of research in place value and decimals without considering how it can be useful to practising teachers, I fall into the trap that many an author of a research work has done. The plethora of research on matters educational with its virtually negligible effect in improving classroom learning situations underlines the extent to which researchers have pandered to the demands examiners and assessment bodies make with consequent neglect of the very issues of learning and children with which their works are concerned. It is as if people who are very concerned with education

- very often teachers themselves - research classroom topics with the best of intentions for effecting improvements there, and then, because of needs of 'passing' are diverted into tortuous paths of statistical substantiations that effectively defeat their original aims. Their resultant efforts are remote, removed and useless, banished to shelves of academics and examiners. Hull (1984, p.13) makes a plea for the employment of the "professional expertise of the practitioner and his perceived problematic" - a more do-it-yourself version of Gattegno's demonstration - but insists that investigation of the teaching aspect of a work of research requires the calling in of other 'classroom practitioner' experts, namely pupils. With due emphasis on the use of the expertise of pupils, I am happy to quote Marilyn Suydam's encouragement to teachers, in her concluding remarks in a booklet on research :

"What better way is there to find out if an idea from research will work with your class than by trying it out with your class? What better way is there to find out that one procedure is better for your children than another than by using it and, collecting some evidence or data?... Do some exploring: become a researcher! This is not a new role for a teacher. Every time you try out a new idea that you've read about or heard someone talk about or have dreamed up yourself, you are, in effect, doing a type of 'action research'" (1978, pp.49/50).

An aim of this project is to present the ideas of research, concerning the teaching of place value and decimals, to teachers without the customary supply of tables of data and formulae as proof of authenticity, leaving verification to teacher and pupil, to establish, if possible. As Stephen Kemmis puts it, "... engaging practitioners themselves in the reformation of their educational practices... could

offer a genuine possibility of changing education from within - something which the curriculum movement of the 1960's and beyond has manifestly failed to do, simply because it has relied on changing education from without" (1986, p.51). Practitioners - teachers and pupils - are in this way adopting research findings of others as their own, more so if, in the process, they have a hand in modifying them to 'local conditions'.

I make no more than passing reference to difficulties researchers have revealed about action research so that, without diluting the enthusiasm which should be part of such ventures, certain hazards can be anticipated, and perhaps avoided. Hutchinson and Whitehouse (1986, p.85) refer to the tension that arises when the 'collegiality, informality, openness and collaboration' that action research 'fosters' (and keep in mind the part Hull's pupils will be playing!) has to "contend with educational institutions that are structured hierarchically with formal asymmetrical relations of power and responsibility" and add that by its very questioning of established practice, action research is seen to be 'subversive' (1986, p.85). How teachers pave the way for their investigations - perhaps by keeping in touch fully with their heads, perhaps by soliciting support of colleagues - is a matter for each practitioner. Assuredly, once teachers and pupils have experienced the thrill deriving from the process of triangulation and the joint reviewing of images of themselves in action, they will be forceful protagonists and loyal users of their own and others' ideas.

My searches uncovered much in the way of ideas about teaching place value and decimals and much less of actual research in the two topics. The ideas, however, presume the recognition and acceptance of the effectiveness of certain general teaching methods.

A persistent and widespread neglect of employment of these methods, however, removes the ideas - no matter how effective or attractive - just one more step away from the act of implementation. I discuss, then, research findings into methods of teaching not necessarily restricted to maths, not to speak of the two topics in question, the aim being that classroom practitioners confirm for themselves their efficacy. On this as basis, discussion of the topics place value and decimals follows.

#### CALCULATORS AND COMPUTERS

"Mathematics is a natural subject to be greatly influenced by technology, especially by calculators and computers" states Hiebert (1984, p.602), who quotes the National Council of Teachers of Mathematics as recommending the following : "Mathematics programs must take full advantage of the power of calculators and computers at all levels" (N.C.T.M., 1980, p.8).

A repeated call for the use of calculators at 'all grade levels', in order to experiment with mathematical ideas and discover patterns, and for the training in skill of estimation, was made in April 1986, in the newsletter of the N.C.T.M. Recommendations in favour of

calculators are the outcome of substantial research as pointed out by Hembree and Dessart (1986, p.83), in their reference to the findings of 79 research reports, assessing "the effects of calculators on student achievement and attitude".

Reporting on research about the effectiveness of calculators in teaching place value and decimals without attending first to the general acceptance of the instrument is like describing the highlights of a certain electric shaver to a tribe of bearded men, who have never questioned the wearing of beards! Hembree and Dessart underline the need for attention to acceptance of calculators when they quote Suydam's figures of "less than 20% of the elementary teachers and less than 36% of the secondary teachers in the United States" as having used the calculator in their maths classes (1986, p.83). This, despite the call for the use of a "device ... too powerful and potentially omnipresent to ignore", encouraging statements about its ability to "facilitate concept development" and to reduce demand on the use of memory, its power to motivate, and its facilitation of "discovery , exploration and creativity" (Hembree and Dessart, 1986, p.83).

Hopeful signs are that more textbooks are featuring calculator exercises, which factor, though not ideal, does encourage the presence of the instruments in the classroom with the implications that that entails for their continued use. Material for lessons like that prepared by Malcolm Swan is discussed later.

The computer comes into its own when used for teaching purposes by engaging children in learning situations that the conventional classroom cannot provide. As a simulator of concrete materials it can "eliminate some of the problems associated with the use of manipulatives in the classroom", making possible the movement of objects, themselves difficult to handle and transforming the tedious time-consuming tasks into facile operations for children. (Champagne and Rogalska-Saz, 1982, p.43).

Champagne and Rogalska-Saz have designed and produced a series of lessons on place value that uses the computer to advantage.

While two hurdles to clear in order to create "a process-oriented mathematizing environment" (Breen, 1984, p.120), are the persuading of teachers in our schools to apply investigative maths activities and the doubtful availability of appropriate software, Breen notes advantages of the mere presence of the computer. He maintains that its implications for the "withdrawal of the teacher from the action and the availability of Logo and Turtle Geometry are factors that contribute to improved learning situations" (1984, p.121).

#### CONVERSATIONS ON MATHS

The interviewing of children has been expressed very aptly by the title of an article in 'The Arithmetic Teacher' (Rudnitsky, A.N. et al, 1981, Vol.28, p.14). The title, 'Talking Mathematics with Children', which hints at a variety of means of engaging children in

conversation, embraces more than 'interview', with its implication of just dialogue between teacher and individual pupils.

The authors, in the article referred to, mention researchers whose findings indicate that

- (i) teachers are in a position to help more effectively when they understand what a child knows;
- (ii) their understanding of children's knowledge can be arrived at very effectively by talking and listening to them; and
- (iii) conversations themselves provide valuable 'instructional episodes (Rudnitsky, A.N., 1981, p.14).

James Hiebert insists that teachers and those concerned with formulating curricula have to find out and take into account children's thinking about maths and children's difficulties with it (1984, p.507). Katterns and Carr (1986) refer to three independent research findings all confirming the advantages of interviewing children in order to "learn about children's computational strategies" (Lankford, 1974), "uncover useful information about their mathematical misconceptions" (Erlwanger, 1975), and aid "individual diagnosis and program evaluation" (Codd, 1981). Using 7- to 8-year olds, Katterns and Carr (1986) applied the Learning in Science Project (LISP), developed by the University of Waikato in New Zealand for using individual interviews to test the quality of children's scientific understanding and to "check on students' mathematical ideas" (p.18). An interesting element of their interview was the 'role-playing task' (p.19), where questions posed to children, like : "How would you help me to understand...?" or "What kind of things would you do to teach



me?" (p.19) served to confirm their assessment of the children's understanding. Their conclusions, although derived from research using the topic, multiplication, did contain a statement of general application, recommending a regular and persistent probing into children's understanding "by having them explain mathematical ideas in their own words" (p.21).

Rudnitsky et al (1981, p.14) distinguish between the child's conceptual and procedural knowledge of maths, the former referring to the understanding aspect of a child's knowledge and the latter to his or her knowledge of the sequence of rules needed for computation. They supply samples of sets of questions one would use when trying to assess and distinguish the two types. For determining conceptual knowledge effective questions might be :

"Tell me. What does this mean?"

"What does this decimal point stand for?"

"Can you explain what a decimal is?"

For procedural knowledge, one could pose questions like :

"What did you do to get this?"

"What is the first (next) thing you think of?"

"How did you figure that out?"

The authors emphasise that the importance of questioning is the amount of understanding that the teacher gains from the child's responses, and not whether the responses are correct or incorrect (1981, p.15).

Considering the very favourable indications for discussion with learners, a strong case can be made for the related procedure of

looking at the errors children make in maths. Besides the aspect of listening to children, analysis of errors would include any means of searching these out - marking tests, examining classwork and homework, and so on. Both Kath Hart's project (1981) and the National Assessment of Educational Progress base their conclusions about children's understanding of maths on the type of errors they have made. Malcolm Swan (1983, p.4) recognises the work done by Hart in underlining the use of "asking the right, probing questions" in order to "discover deep misconceptions", adding that only by exposing misconceptions can teachers know what questions are "worth asking".

#### GROUPS

Attempts to afford all members in a class meaningful experiences in maths should involve the teacher in activities, with three aims in mind, according to Schoen, in his research project (1986, p.44). Firstly, activities for learning must not sacrifice quantity and quality, when one attempts to cater for all pupils, states Schoen. His second condition is that built-in opportunities for getting to talk to all pupils have to be provided for, and thirdly, he insists that enough direction should be given so that no pupils are left, at any stage, not really knowing what to do (1986, p.44).

In the same article, strong indications of the benefits of small-group teaching are reported to be the findings of a study by Noddings (1985), where the advantages of small groups are listed as: the sharing of 'background information', the 'challenging one another's

thought processes, and the 'helping each other develop new cognitive structures'. Peterson and Janicki (1979) compared the effects on children's learning in large groups and small groups. The work done by children in their research in the large groups was comparable to the individual work done by pupils in the normal size class, while the small group afforded ample 'student-to-student interaction' (p.21). The findings that the able pupils fared better in small groups and the weaker pupils achieved more in traditional class-type situation, differs from the research of Noddings (1985) and others. The abstractor's comments, however, are significant, pointing out that weaker pupils might just not have featured in the small groups because activities had not aimed also at engaging them in thought.

## INVESTIGATIONS

Since children from very early on all use inductive reasoning - the 'key to the investigative process' (1986, p.36) - Borenson recommends teachers harnessing this thinking by setting investigations in maths. He tells of a lesson he organised as an investigation, describes the effects it had in encouraging the sharing of ideas amongst the pupils and notes how pupils had come to realise 'that the pursuit of mathematical investigations was not only within their ability but that it was an exciting process as well' (1986, p.38). This method of 'constructing mathematical knowledge' (Gordon, 1978, p.252), calls to mind Marshall Gordon's view of maths as a personal creative activity where one comes to grips with the wrestling that went on in the production of an item of maths. For the interest to be there, he

maintains an important requirement of the activity of 'personal construction of knowledge' should be that the learner acts as 'problem poser/chooser' (1978, p.263).

In the section on decimals, a description of the work of Swan illustrates how the investigative approach is applied to the topic.

#### MANIPULATIVE MATERIALS

Jerome Bruner (1966) states that there are three systems by which man represents reality: action, imagery and symbol. Hynes' definition: "Manipulative materials are concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved..." refers to materials that are available for children to use as distinct from demonstration models (1986, p.

The use of materials has had the support of learning theorists for some time. Kennedy (1986, p.6) points out how William Brownell as early as the 1930's spoke of meaning theory which replaced mental discipline and stimulus - response theories. For permanent learning, according to Brownell, children had to understand the basic concepts of that learning, states Kennedy (1986, p.6). It was his theory that awakened interest in the use of manipulative materials (Kennedy, 1966, p.6). Kennedy explains further how the studies of Piaget (1952) and more recently, for example, Skemp (1982) led them to conclude that children pass through stages in growth of cognitive development, in all of which manipulative materials are needed. He refers also to

Dienes who is a protagonist of "an abundance of materials" (1965, p.10), and who, in 'Modern Mathematics for Young Children' insists strongly that the materials "described in this booklet are not intended for 'demonstration' by the teacher but as an essential tool in the learning armoury of every child" (1965, p.10). Dienes accuses maths teachers in general that they "grossly oversymbolize", adding that "a series of events is a far better teacher than a series of explanations" (1965, p.14).

In his studies described in 'The Elementary School Journal' of May 1984, James Hiebert cites researchers like Behr, Wachsmith, Post and Lesh (1984) in support of the use of materials to bridge the gap between "real-world objects" and symbolic representations, and he concentrates on recent research, stating that that conducted earlier looked more at achievement in mathematical operations rather than meaning and conceptual understanding.

Hieberts (1984, p.507) concludes that research indicates that, considering the major distinction between form and understanding in maths learning, success has been achieved in teaching form at the expense of understanding, illustrating this by describing how students can "memorize and follow rules" (p.507) yet not make sense of them, and consequently are unable to apply them. Referring to the findings of Scott (1983, p.61) as one of several confirming that poor use of manipulative materials was being made, Suydam, in 'The Arithmetic Teacher', February 1986, deplores the widespread omission of the use of manipulative materials in classrooms.

As in the case of calculators, hopes are expressed that activities designed round the use of materials, like the 400 learning activities of Richard Skemp, that M. and B. Harrison (1986, p.60) mention, will induce the use of materials, while textbooks are recognised as concerning themselves only with 'pictures and symbols'. And in support, the Harrisons maintain that experience with physical objects, discussion about it, the use and recognition of pictures of that experience, and - only then - symbols, as a record of it, ensure sound development of mathematical abstractions (1986, p.60).

One needs to be conscious of teachers' reluctance to use materials, so that when reference to the employment of blocks, rods and other apparatus in the learning of place value and decimals is made, sufficient persuasion about just the idea of using materials and recommendations about acquiring them, are given. Hilary Shuard (1984, p.605) insists in an article in 'The Elementary School Journal' that more than a textbook is required today, mentioning materials, amongst others, as indispensable adjuncts.

#### PLACE VALUE

It is generally accepted that place value forms the basis of the understanding of decimals. Kath Hart's C.S.M.S. project (1981), where tests in mathematics on many children in the junior secondary phase were set and analysed, has a hierarchy of items in the tests with place value, dealing in whole numbers, preceding the decimal work

with tenths, hundredths and thousandths. Earlier research, based on the analysis of errors children made, confirms that an understanding of decimals is built on a basis of place value in whole numbers (Carpenter, T.P. et al, 1981, p.34). A test item in the study by Carpenter et al had 40% of the children tested, at a loss, about which of 0,037; 0,37; 37; 37 000 was 37 thousandths. Granted, as the researchers concede, the fact that many children responded with 37 000 could indicate confusion between thousands and thousandths; they point out, however, that in items where zeros appeared to the left of other digits uncertainty was found to exist, indicating that understanding of place value was faulty (p.35).

In earlier research, Carpenter and different associates found that 9-year-olds reversed digits, in many cases, when confronted with subtraction of two digit numbers where the units being subtracted were more than the other units (1975, p.653).

Reporting on the Third Mathematics Assessment he (Carpenter et al, 1984, p.487) focused on place value again, noting that only 73% of the 9-year-olds had understood that 70 represents 'seven tens'. S. St.John Jesson (1983, p.69) quotes research of McIntosh on two separate occasions, in 1978 and 1979, where little success with subtraction or multiplication involving place value skills was achieved. Ward (1979, p.78) sums it up with the statement, "By the age of 12, 15% to 20% of children were deficient in applying place value skills overall".

There need be no doubt that what the National Council of Teachers of Mathematics describes in its Yearbook of 1983 as "a wonderful element making possible the naming of any number as large as one likes or as small as one likes, using only the digits 0 to 9" (p.128), is, short of being appreciated by the majority of children, actually just not being understood.

The beauty of the power of place value should be utilised. This should be one aspect of our approach to learning place value. For a second aspect, I refer to a finding by Tucker, who examined children's errors in addition and subtraction of two digit numbers (1981, p.24), maintaining that errors occur when children perform arithmetical operations that are dissociated from physical operations. In 1960, Dienes (p.70) was arguing for the inclusion of 'structural support' in teaching place value, suggesting later in 'Modern Mathematics for Young Children' (1965, p.54) that the idea of power, on which place value depends, should be experienced by children in games, where, for example, on the first day of a school term, say, they are asked to bring two other children on the next day, and on the day after, each threesome is asked to bring another two groups of three, and so on, for base three. He describes the same activity, for experiencing different bases, and then recommends the use of materials, like counters, blocks, straws, etc., and games with dice, before teaching symbols. Mental imagery needs to develop from physical operations, according to Tucker (1981, p.24) if children are not just to have to remember what the teacher said. Richard Skemp's 'relational' as opposed to 'instrumental' learning (1979,p.46) comes to mind, when



researchers like Dienes and Tucker advocate experience in activities with materials as a first step, saying the number next, and, only finally, recording it as a symbol, in attempts to establish understanding. Cockcroft (1982, p.87) notes the need for the understanding of place value, and also urges teachers to use 'structural apparatus', mentioning the abacus and stressing that mental and oral work had to precede written work (p.92). Hollis (1964) did research on the use of Cuisenaire rods compared to a traditional approach, finding : (i) "that pupils taught by the Cuisenaire-Gattegno method acquire mathematical concepts and skills that were not taught in the traditional program"; (ii) "the high ability level pupils benefited more from the Cuisenaire-Gattegno method than did the pupils with low ability levels", and (iii) that, in general, "females did as well as their male counterparts" with the rods. One of his recommendations was that "Teaching materials similar to the Cuisenaire materials should be utilised in concept development in the first grade mathematics program" (Hollis, 1964), which refers to the very early years, when children in our schools in Sub B are introduced to place value. Cuisenaire rods, Dienes blocks, all sorts of counters, pocket charts and the abacus are indispensable, then, for the junior school teacher.

Lindquist (1984, p.601) cites an example in the use of materials, stressing that oral work should accompany or follow concrete experience, by suggesting that a whole number be expressed in many ways: for example, 243 not only made up of 2 hundreds, 4 tens and 3 ones, but also 24 tens and 3 ones, or 243 ones.

Finally Kath Hart (1981) found children having a superficial knowledge of place names that "more probing" soon revealed to be weak (p.49). Numbers over a thousand, according to her findings, were proving difficult even for children of average ability, and test items with zeros as place-holders caused problems.

In Hart's words, "It is above all clear that the learning of whole numbers and decimals is not just a matter of recalling some place-names and a few rules of computation, as it often appears to be from the text books" (1981, p.64). Hiebert's notes - referred to earlier - about form and understanding are confirmed by Hart's concluding observation that those children who did know the rules often applied them incorrectly (1981, p.64).

#### DECIMALS

The most significant problem in children's work with decimals is their inability to order decimal numbers. According to Hiebert (1984, p.505), a number of researchers (Carpenter, Corbitt, Kepner, Lindquist and Reys, 1981, p.34; Ekenstam, 1977) have recorded the occurrence of this difficulty. Anne Grossman (1983, p.32) notes the misunderstanding concerning decimals and the correct ordering of them and recommends that attention be paid to the concept of equivalent decimals by the use of zeros to make decimals equal in the number of places they have to the right of the decimal point. Her suggestion is sufficient help for pupils who basically understand decimals, but

for children with fundamental problems, Hiebert (1984, p.505) sees the need to understand decimals as representing "the integration of part/whole concepts of fractions and place value concepts of whole numbers". The two approaches of Carpenter et al (1981, p.36) are the building of a sound understanding of place value and the connecting of the idea of decimals with that of common fractions. Of the two, Hadar's remarks about research in Israel in 1985 where the teaching of common fractions with denominators 10 or power of 10 served as introduction to decimals, lay emphasis on the linking of common factors and decimals. This makes sense when the use of materials is considered, because where objects can illustrate the place value concept for whole numbers and can be used to demonstrate parts, they fail to illustrate numbers to the right of the decimal point, unless these are linked to common fractions.

The survey of Peter Foxman's APU project mentioned earlier serious misconceptions about decimals :

- (i) "the longer the decimal, the smaller the number and vice versa"; and
- (ii) "decimal numbers can be treated as though they are whole numbers". (Foxman, 1985, p.19).

Peter Foxman's research supports the conclusions of Ekenstam (1977) who - Hiebert writes - found that "most students do not connect their understandings of fractions with the decimal symbols" (1984, p.508). Models, according to Piaget, Inhelder and Szeminska (1960), act as the bridge between the ability that children have already developed, "to

partition continuous lengths and regions, such as sausages and pies" and the concept of decimals, writes Hiebert (1984, p.504). Hiebert then advocates that "frequent, explicit links between the physical and symbolic representations" are practised.

If true teacher involvement in the implementation of improved teaching methods in decimals is to be ensured, relevancy must be a prominent factor and processes of action research apply here. A particular teacher's investigations of his or her class's difficulties should be the starting point. Then, using suitable materials, teacher and pupils should jointly devise strategies that answer their needs and that lead to learning with understanding.

An interesting study based on the work done by the CSMS team of Kath Hart was started in 1983 at the Shell Centre for Mathematics Education at Nottingham University by Malcolm Swan. Using the analytical approach of Hart's project in 1981, Swan adopted a 'conflict' teaching process in which a set of pupils were to participate in discussion and reflection on their own misconceptions and errors (p.1). A comparable set of pupils, in a 'positive only' process, were provided with "simple and understandable" methods (p.1) and required to do work with decimals and place value, trying as far as possible not to make any errors. The contents of both approaches were based on "specific areas of known difficulty" obtained from a "diagnostic pre-test" (p.1).

The conflict group underwent a destructive phase initially in which

old ideas were seen to be inadequate. Pupils then reflected on, and discussed, in groups, the different answers they had obtained, becoming aware, in the process, of the need for effective methods, which they proceeded to evolve by discussion. Their corrected methods were then used and practised. Of significance was their creating an awareness of their need of new concepts and effective methods. The positive only approach took no cognition of errors, the purpose being the teaching of simple successful methods from the start by the avoidance of errors as much as possible. The 'simple successful methods' also featured in the 'conflict' group in that, after the pupils in this group had answered questions initially that exposed common misconceptions, a 'more reliable method' was imposed by the teacher, and they answered the questions a second time using this method. In the 'conflict' case these methods were simply applied without any explanation, in order to obtain the correct answers.

After observing all appropriate research constraints, Swan's conclusions based on the test results were that :

- (i) both approaches helped pupils understand decimals and place value;
- (ii) the 'conflict' approach was significantly more effective in 'permanently' eliminating and correcting understandings;
- (iii) no pupils suffered because they were confronted by misconceptions that were new to them (p.1).

On the strength of the research, Malcolm Swan has designed a 'Pilot Version' resources book, 'The Meaning and Use of Decimals: Calculator Based Diagnostic Tests and Teaching Materials'. The book employs the investigative/group-effort approach. Using materials and

illustrations extensively, the activities encourage pupils in discovering answers to questions like the following :

What are decimals?

How are decimals used?

What common mistakes should be avoided?

What do multiplication and division mean in a decimal's context?

How can I decide what buttons to press on my calculator?

How can I interpret the answers given by my calculator? (p.1).

Earlier studies meriting the attention of teachers who need to verify them in the context of their teaching/learning situation are worth noting. Faires (1962) and Willson (1969) investigated the outcome of teaching decimals before common fractions. While Faires found that decimals developed logically as an extension of place value, did contribute to increased computational achievement and to equal understanding in relation to the traditional sequence, Willson did not detect any "significantly greater gains".

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# RUNNING REPAIRS IN STUDENT/LECTURER CONFLICT

## A STUDY IN SELF-HELP USING ACTION RESEARCH

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Action Research starts with an 'identification and clarification' of a General Idea, in the words of Elliott, and the 'Idea' he defines as 'essentially a statement which links an idea to action'. (Elliott, 1981, p.90). On being approached by a deputation of students from the B.Prim.Ed. Maths class, Chris recognised the possible germ of a General Idea for an Action Research process.

Chris had offered part-time lecturing work to post-graduate students, and Ed, taking it up and lecturing maths to the B.Prim.Ed's, found himself the object of criticism. Students objected quite strongly to his approach, so much so that representatives were sent to Chris to voice the class's objections. Obviously, immediate action was necessary: instruction in maths was about to cease for more than thirty students, and Ed's teaching contract was in jeopardy.

Fully committed himself, Chris was in no position to have to take on extra teaching. He took the bull by the horns, using the opportunity to initiate an action research venture in the situation. There were M.Ed. students just looking for opportunities of applying action research, and the offer to take on the project was put to them. Chris first approached Ed. As the lecturer, he was feeling disconcerted about classes that had turned sour, and as an M.Ed. student, work for him was piling up and needing attention.

Instead of calling him aside, imparting some tips on lecturing techniques to university students - a step which reduced students to

objects to be 'handled' and demeaned Ed's position as lecturer - and then sending Ed to resume the classes, Chris offered him the opportunity to apply action research to the problem. Ed was to participate on equal terms with a team of student volunteers from the class, in a co-operative effort to redeem the situation. I was invited to direct, co-ordinate and facilitate the research process. Also an M.Ed. student, I was glad of some direct involvement with action research, and while a complete outsider as far as the students were concerned, I knew Ed well. The fact that we both lectured at the same college and that Ed knew of my encounters with classes of students in similar problematic situations, helped him accept me as one likely to understand the situation and as one unlikely to assume any superior stance or to exploit the position that he was in. At the same time, I was virtually a disinterested participant in terms of the outcome. I could expend all effort on making the process work smoothly without feeling threatened by the direction proceedings might take.

Prospects were exciting, possibilities for action on real issues that, as Hull put it, "had meaning in the context of the classroom" (1985, p.88). The three advantages of such research, that Hull quotes from a study of Bennet and Desforages, are relevant :

"Firstly, the results are more likely to be used if the consumers are involved in the research process. Secondly, such involvement is more likely to lead to the complexities of classroom life to be recognised, and thirdly the time lapse between research and implementation will be markedly reduced."

(Hull, 1984, p.6)

Rejecting the temptation to devise with Ed some unilateral solution to impose on the class, Chris made an offer of power to the students and Ed: the process of negotiation to improve their lot was to be in their

hands. He was, in Hull's words, creating an "interface that counts, ... that between practitioner and pupils" (Hull, 1986). What counted even more here was that the cause of the problem, Ed - according to the students, - was himself to be in discourse with them about their difficulties. It was to be, as Chris pointed out to the class and to the students who volunteered to form a committee, true collaborative research, where, as Hull states, "participants become co-authors of liberating action by way of communication, through dialogue in which all participants are both learners and teachers and through which critical analysis of practice is developed" (Hull, 1984, p.7).

The question was to see now whether the offer to the students afforded them a real voice in deciding what to do and how learning should proceed. Such an offer, Hull maintains, has pupils (or students) 'assuming responsibility for their learning'. (Hull, 1985, p.7). With about three lectures already behind them;, students were anxious not to waste any time and a process that offered immediate steps towards restoring proceedings intrigued them.

My first investigations - besides frantic searches through Action Research books - was to find out what the complaints of the class were. Chris's meeting with the students, before their next lecture with Ed, where he responded to the student deputation was informative. (He had lost no opportunity: his proposals to the class were being recorded!) Apparently the class was finding Ed too much of a contrast to Chris, to whose lectures they had grown accustomed. Ed was talking too much, they found themselves doing too little work, and in general they seemed disgruntled by the switch.

One could see reasons for their attitude - roughly, two. Knowing Ed as I do, dissatisfaction of the students with his approach was understandable. I supposed, however, that there was more to it. Were the students not also disappointed that Chris was not taking them as expected? A feeling of rejection could be aggravating their annoyance about presentation of the lessons. There was the feeling that they had been brushed off, given a raw deal. An obvious solution would be - as far as the students were concerned - a take-over of lecturing duties by Chris.

If this was expected, the response must have come as a surprise to them. Chris acted as a true educationist. The people affected by, and therefore directly concerned with, the research process were to direct it. Ingvar Werdelin (1979, p.10) states that such a situation constitutes the right ingredients of Action Research, and Hull (1986) maintains that proposals offering such a high degree of empowerment to the students themselves implied a caring attitude on the part of the person in charge. Here, for the students, was their lecturer offering them the means of

- (i) designing their lessons;
- (ii) having them recorded on video and audio tapes; and
- (iii) having these tapes at their disposal to examine and analyse in order to gauge what was effective and what not.

The class's feelings of rejection were dispelled. To the volunteer group who were to collaborate with Ed in designing the class' lessons, was offered the bonus: an opportunity of improving their teaching skills.

Action research, by nature, according to Werdelin, is 'an educational process', and 'a means of taking action for development'. (Werdelin,

1979, p.10). The B.Prim.Ed. research venture was pregnant with possibilities of improvement on a number of issues. First, was the obvious one of resolving the problems experienced in the class; second, Ed as a lecturer stood to benefit; third, principles of teaching and learning would form the topic of most principles of discussion and fourth, the Action Research process would be experienced, refined and, hopefully, be seen to succeed.

Once the student volunteers had indicated who they were - there were six of them - Chris went to work behind the video camera and the process was under way. A tape recorder had been on from the start when Chris had introduced it all. Henceforward lectures and committee meetings were to be recorded either on video or audio-tape, and sometimes on both. Three lectures and three committee meetings served as data sources and Appendix A lists these in order, giving tapes and counter numbers that are relevant. Transcriptions of extracts from the tapes appear on six Analysis sheets, one for each occasion, with the exception of one. Here, significant items of data are recorded in the sequence they occurred, next to the issue to which they refer. The Analysis sheets were used to compile notes on progress in each of the four issues and I proceed to deal with these.

#### Resolution of Class's Problem

While there was more to it than mere dissatisfaction with Ed because they had been 'foobbed off' with someone other than Chris, just the time and attention that Chris had devoted to the students already had had an effect. Both video and audio recordings show the groups that Ed had arranged the class into busily engaged in serious discussion on

the topics at issue. Of course, one bears in mind the students' awareness that camera and tape recorder were directed on them. Nevertheless, their complaint had been heeded. Chris himself had stepped in to help and feelings had been smoothed over.

Ed, however much he took pains to indicate that he felt the complaints had been unreasonable and that the measures they were taking now were exaggerated and quite unnecessary, was careful about the instructions he gave, was considerate of the students' need of seeing the transparencies and did confine his talk and movement. So his approach from the outset contributed to the prevailing atmosphere of industry of that first class so that Ed's tentative, contrite suggestion at the end, that perhaps the lesson had met with more approval than previous ones, drew forth even some murmurs of assent!

Transcriptions of subsequent lectures reveal significant and dramatic improvement in class proceedings and in interchange between Ed and the students. We shall trace the development in Ed's lecturing technique and approach in the next report.

#### Ed as Lecturer

A new air of quiet assurance, in contrast to the insecurity and lack of confidence of a bombastic, aggressive Ed was clearly evident in the confident tone and relaxed movements seen in the video tape of the last lesson of the action research programme. Instrumental were the thought, intense discussion and detailed preparation characteristic of



the meetings of student team and Ed, subsequent to, and preceding, each lesson. The viewing and analysing of each video and audio tape together formed, as it were, the chipping, sculpting, moulding and polishing process of the repertoire of Ed's lecturing techniques, while the classes served as test track where ideas and strategies fashioned in the workshop were tried out.

It was no easy process. Starting with pieces of broken relations that we were to restore, our very tool of reform had to be a viable relationship for co-operation between the parties. This is what action research is about: developing relations which themselves form the basis of the process of negotiation.

The viewing together and ensuing discussion round moments of surprise, amusement and suffering had a bonding action that coincided with Hull's assurance: "The images liberated a critical discourse by way of which a degree of shared experience was negotiated that could form the basis for shared planning". (1984, p.18).

The opening video scenes marking the start of a lecture along the lines of a dramatic new research programme would have had even the most fervent devotee of its powers beset by a battalion of misgivings. Ed is seen at pains to show that he regards it all as quite unnecessary. Exaggerated gestures and heavy emphasis on operative words all drive home the point that Ed is not approving. Audio and video tape extracts can be seen on the Analysis I sheet. Displays alternate between demonstrations of being deeply offended to shades of

the old casual, offhand manner - the attitude that the whole business is hardly worth worrying about. "... if you picked up anything at all ..." conveys the criticism of a body of casual, couldn't-care-less students.

Perhaps the initial reaction is necessary; the old Ed is not going down without a fight. The scene changes. Group activity commences. Having given vent to the irritation and wounded feelings, Ed is seen giving clear, precise instructions in a quieter mood. Movements are relaxed. He seems to have heeded criticism. Granted, the students do appear to be engaged in the set tasks and their industry could be contributing to his change. But one must not underestimate the effects of self interest - and I mean this in a positive sense. The participation aspect of action research is a secret of its power. People at the design-end show great interest in the manufacturing and testing of their article at the other end. When designers are also the craftsmen and the users of the product, keen, committed interest prevails. Ed's simulated air of detachment - a cause of the breakdown - cannot continue; his actions in the class now are the outcome of his planning. True, he shares in the planning but his partners now have interests in the lesson outcome also.

A give-and-take procedure - with, admittedly, interruptions of destructive behaviour from both sides on occasions - characterizes the Ed/student team workshops. Effects of their joint preparation are seen in the lesson following the initial one, the transcription of extracts of it appearing on Analysis IV sheet. Ed's quiet authority

indicates confidence. He achieves excellent rapport with the students, and constructive discussions in which many students participate are the first fruits of the planning and design that went into the preparatory workshop.

These and subsequent workshops and the classes they cater for, extracts of which can be seen on the Analysis sheets, are recorded in full on tapes which are included with this project. Analysis sheets V and VI are transcripts and notes of extracts only from two of these sessions: the lecture of 19 May and the meeting of Ed and the team - the last one - on 21 May. Extracts on these last two sheets underline the degree of meaningful discussion and improved relations that has been achieved - tribute in this context to a research process that still awaits due recognition and full implementation in the field of education.

#### EXPERIENCE IN EDUCATION

With the improvement in relations among people and conditions, go the acquisition of knowledge about teaching, development in action research skills and growing appreciation of the power and virtue of this instrument of change in education. These valuable items - extras, in a way, of the main aim of the process - add to the educative force of action research and make it so attractive for the world of teaching.

Discussion on teaching points that emerged continually played a

two-fold role. The concern about teaching was evident by the intense discussion occasioned by the appearance of problems concerned with method, approach, control and so on. Opinions reflecting valuable insight were exchanged and quite a few fears and uncertainties could be dispelled by them.

That represented one outcome. The second was less obvious. Behind the discussions about teaching the critical examination of Ed was going on, and although such scrutiny was fulfilling the main aim - still it did not detract from the benefits accruing to the fund of teaching knowledge.

The workshops where Ed, the student team, I, and occasionally a video camera operator, reviewed and planned together were naturally the most productive with reference to the action research input, because here the previous lesson was discussed and the various learning conditions were examined. Significantly, the students are seeing themselves in action. Hull's statements concerning the teaching force of video apply. His descriptions of video images - "a richer representation of data ... significant for the understanding of ourselves ... incisive in its ability to impact us" (1986) - come alive for us. We begin to share Hull's enthusiasm for video's role as facilitator when he writes:

"Sharing with a video image communicates surprise to one another. Everyone feels exposed, resulting in discourse about the practice on a level of integrity that is ... conducive to meaningful exchange."  
(1986)

As relations between Ed and the class improved and the students found they were achieving more in their classes, emphasis in the talks between Ed and the team shifted from analysis of Ed's role as teacher to preoccupation with the problems of teaching itself. Fruitful discussion covered a wide area of teaching - this activity over and above the deep reflection now taking place during the improved classroom activities. The student team were being exposed to the class discussions again during their workshop viewings, and they were enjoying the experience!

In the video of the last meeting of the team and Ed, one observes students engaged in deep, insightful discussion on group-teaching and teaching in general. What of their views on action research? Can these 'powerless people' - in effect, considering the situation they started in - see results of their involvement in the research process? Was power given to them? Ingmar Werdelin, on participatory research, states that people involved in it become more self-reliant and can change situations because they have been sensitised to their own strengths. (1979, p.11). Statements from the last video demonstrate, without any doubt, the students' positive impressions about action research.

## THE TECHNIQUES OF ACTION RESEARCH

### Skills of Negotiation

The first meeting of Ed and the student team - see Analysis II sheet - is significant. The process and rules of negotiation are to be

established. The operators of the negotiation process consist of a group of dissatisfied, angry students and their lecturer, Ed, who has been the target of strong criticism. The main aim is the creation of workable teaching/learning conditions which are to be wrested from the remains of a collapsed relationship. The task is a daunting one considering it is to be a do-it-yourself rescue bid in true action research style.

However, participation is the key factor in action research; it is the direct involvement of all concerned in the issue at stake that gives action research its strength. Chris is a non-participant but is there at the start to help us all take the first few corporate steps. We have the video tape of the previous lesson and commence proceedings by gathering round the screen to view it.

Selections from the tape appear on Analysis II sheet and show Trevor of the student team essaying a first tentative criticism about a point of Ed's presentation. Ed replies. Chris listens and waits. The next criticism from Riva occasions a deft parry from Ed that effectively puts her down. Politely Ed reprimands her for not having listened. Then Chris launches out. "You must thump him!" he tells us. A stir of surprise from all concerned. The message, however, is received loud and clear. Negotiating parties meet on equal terms.

Authority Ed derives from his position as lecturer is not operative here. If the collaboration is to be real power resides in everyone, as a negotiator, equally. The aim of the 'shared experience' of video viewing leading to 'shared planning' is in process of being

attained. (Hull, 1984, p.18).

There is more to Chris's mock outburst here. He is setting down counter tactics for us, for all the negotiating that awaits us - preparing and equipping us to cope with a doughty opponent. We are being warned that we're not attending an old ladies' tea party; we're not to pull punches.

Subsequent discussion in that meeting never equals the import and dramatic moment of that first exchange, but a quiet give-and-take action is seen to develop, Chris skillfully guiding proceedings.

Extracts from the audio-tape of the next team meeting with Ed (see Analysis III) portrayed attitudes and developments that could have spelt the collapse of the whole exercise. For anyone expecting to build straight on from where the last meeting ended, this meeting comes as a shock. The improved relationships and benign attitudes are not as established as they appear to be. But as attitudes reside in people - mercurial, inconsistent beings that they are - how can anyone stake one's expectations on their constancy?

The old couldn't-care-less attitude is evident in Ed's remarks about the next week's lesson. Perhaps Ed is not feeling so good. Perhaps these students are also starting the day badly. Reactions to Ed's take-it-or-leave-it approach is equally negative. We are caught in a degenerative spiral. One wonders what the end of such a nose-dive would imply for the research process. Fortunately we are left

wondering because of a timely rescue bid by a student called Trevor. His interruption of a series of statements and responses that ring the death knell of all that has been built up so far has startling effect. Things look about as black as they could when Trevor pipes up with, "I think that's most exciting. You see ...". Dramatically the situation is saved! Immediately there is a complete change of tone. One hears Ed and students involved in constructive discussion. It is surprising how one positive remark - one statement of hope - discounts and neutralises all the destructive forces. How much can one attribute such a quick retrieval of former levels of attainment to the forces of co-operative effort already established by action research? Can the accumulation of positive regard for one another, occasioned by the shared viewing, be so strong so soon?

It is worth noting that the progress achieved in the construction of positive attitudes in one meeting does not necessarily follow on up at the start of a subsequent session. Like feelings, attitudes vary unpredictably. But the bonds of friendship or co-operation that the sharing and collaborative action build is cumulative. As participants get to know one another, the depredations that temporary fretful feelings and bad moods can cause are withstood or eliminated more effectively. The process of negotiation is less likely to suffer serious breakdown the more sessions together a team has behind it.

Recordings of the last two events - a lecture and a final team meeting



- confirm this. Shared experiences have strengthened relations to a degree that affords effective insurance against damage to which slips, omissions and potentially harmful statements might expose negotiations. The spin-offs of increased sensitivity to one another and also of a more skilful use of negotiating skills contribute to the recovery.

The brief reference that the role of the topic of teaching had as a facilitator in warding off disruptive effects of criticism deserves mention here.

As a topic for discourse problems of teaching arising from discussion effectively divested criticism of its sting so that its effect was only constructive. Ostensibly concerning ourselves about matters educational students could criticise Ed's approach or attitude and find Ed accepting it without resentment.

#### RECORDING EQUIPMENT AND PROCESSING OF DATA

"The initial difficulties of working with video equipment are clear",

states Charles Hull.

"First, there is a need for a period in which teachers and pupils can overcome their narcissistic awareness of the camera and become comfortable in its presence."

(Hull, 1985, p.92)

To a lesser extent this applies also to the tape recorder. One

detects a growing familiarity of the student team with the presence of video and audio recorders over the period of research. Quoting Hull again,

"As this investigation shows, recordings of performances are a rich medium for the development of analytic and reflective critique."

(1985, p.92)

The reference in the last quoted extract is to video recordings. Often criticism can be misinterpreted when it refers to a lesson, and here the video can recall - as many times as one wishes - the lesson, and the particular point of the lesson in question can be considered in its context. Faulty memories, speculation, argument and misinterpretation are eliminated with the use of video recordings. A case in point is the irrefutability of the students' contention about Ed's rapid showings of transparencies during that first recorded lesson; the evidence was there for all to see including Ed. As a teacher in Hull's action research team put it: "I have questioned my teaching much more and consequently changed and adapted certain aspects of it" (1985, p.95). The off-hand manner of Ed which annoyed students could be looked at and the different expressions that convey the attitude were studied. As Middleton explains, "One incident can be frozen and replayed for indepth analysis again and again", and "small nuances - gestures - smiles - indicate attitude and aid in analysis" (19.., p.34). Clearly the amount of movement, gesture and talking of Ed varied inversely to his degree of confidence in whatever he was doing. The later tapes of successful, well-conducted lessons depict a serene, still, silent Ed!

For shots of students a wide-angle lens is recommended by Middleton (19... P.34) to prevent selective recording. Difficulties are encountered when a class is engaged in group work. Unless the video camera has a radio microphone, or each group has a tape recorder(!), the video record seems to serve little function. Then a tape recorder comes into its own for the effective record of a group's discussion. Our group lessons used the video camera for Ed's teaching time and for report-back sessions, and the tape recorder during group discussions.

Synchronising video tape with audio tape can present problems. Audio tapes need 'markers' to indicate when the tape recorder was switched off at the end of a discussion and when recommenced for the start of another, after a class report-back, during which the video had taken over. A cough, or bleep or blow into the microphone is necessary to punctuate these stops and starts. Again, a radio microphone is useful when the lecturer or teacher joins a group that is not the group with the tape recorder.

As a source of data, video recordings, according to Hull, are the ideal from the point of view of qualitative research. He finds the richness of it lessens the risk of unsuitable selection. Rather, he recommends that it be 'cut down to size' by editing, because it is so rich, the data being so much. He advocates playing a tape a number of times to see what interests one, suggesting one viewing without the sound on in order to highlight the non-verbal 'language' which he explains reveals attitudes, and to which I refer at times on the tapes

of the project. (Hull, 1986).

In conclusion: This record in itself pays tribute to the positive manipulative power of action research in bringing about improved conditions for education. All recognition in this case, however, cannot go to the research process; students and Ed played a significant role. I suppose enthusiasts could purport this again to action research's unique powers, but insistence in reference to qualities of Ed is necessary. Ed's sustained willing participation in this whole exercise must have endured severe attack at time, when criticism - not always intentionally - was particularly heavy and when clumsy handling of the negotiation occurred. A magnanimity was evident on these occasions - a nobility of soul - that makes Ed the real hero in this story and that contributes truly to the reputation of action research and ultimately to the advancement of education.

Is that it then? Now that aims have been achieved, are records filed away and teams disbanded? Is it business as usual? Action research is a commitment to developing relationships. Its working strength is improved relations from the fellowship of co-operative ventures. It serves education that has to be continuously reaping benefits. Once started, action research needs to keep going, not at the intensity needed for times of emergency, but nevertheless seeing to the upkeep.

In our case, I would suggest monthly meetings take over the weekly ones, Ed and the student team maintaining contact and - when deemed necessary - making a video record of the class for their analysis.

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APPENDIX A : MEETINGS AND TAPE REFERENCES

ANALYSIS	DATE	VIDEO TAPES	AUDIO TAPES	NATURE
I	M 86-03-24	-  V, 0000	A, I 000  197 to end	Intro: Action re proposals put to class  Lecture
II	W 86-03-26	-	A <sub>2</sub> I 000 to end	Committee, Ed and I, with Chris getting process under way.
III	W 86-04-02	-	A <sub>3</sub> I 000 to end	Committee, Ed and I.
IV	M 86-04-07	  V, 1836	A <sub>4</sub> I 000  A <sub>4</sub> I 381 to A <sub>4</sub> II 142	Ed introduces Sue (video) and me to class.  Lecture
-	W 86-04-09	-	A <sub>4</sub> II 142 to end	Committee, Ed and I.
V	M 86-05-19	V, 2994 to end  V <sub>2</sub> 0000 to 0380	A <sub>5</sub> I 000	Lecture
VI	W 86-05-21	V <sub>2</sub> 0408 to end		Committee (last meeting)

	AUDIO		VIDEO		
	COUNTER		COUNTER		
IV Action Research	A <sub>1</sub> 000 to 197	C. 'By having it' on record one's able to explore that a little better.  C. How many of you would be keen to be involved in that?			These remarks form the proposals and signify the groundwork of the action research process. (In answer to query about video's presence) Chris puts forward proposals in response to class's protest about Ed., which Chris has taken seriously).
II Ed, Lecturer	A <sub>1</sub> 1 197	(Instruction to groups before starting).	V <sub>1</sub> 0000		'You protested; <u>you</u> may do something about it' (He cautions that it will involve time. Six students volunteer to form a Committee).
	A <sub>1</sub> 1 215	Ed: "If you picked up anything at <u>all</u> ."	V <sub>1</sub> 0007		Humility? Doubtful. More, 'From the way you complained I don't expect you to have learnt anything in the previous lecture.'
	A <sub>1</sub> 1 218	Ed: "... and I'm not going to <u>tell</u> them." (Guarded).		Grins, pulls a face; head down, hand and arm stretched out.	Meaning he's not prescribing to them. At great pains - and showing it - not to offend. In other words they're <u>so</u> sensitive they've got to be handled with <u>kid</u> gloves. He's annoyed. 'Make no bones about how <u>careful</u> I am being'. Smarting.
		"Right, OK" (Pardoning) "Tell me if I'm taking them off too quickly".			Not at <u>all</u> OK. He's offended.
	A <sub>1</sub> 1 291	"It's interesting to <u>me</u> ; may not be interesting to <u>you</u> ."  "Now to ensure that we do <u>some</u> work this morning." "Put that word 'must' in inverted commas." (Aside)		Much movement and gesture.	That was <u>your</u> opinion in your complaint; I'm going to have to be told then.  I'm not imposing my views. please!
II Ed, Lecturer I Class	A <sub>1</sub> 11 000	(Instructions clear, precise). (Groups at work).	V <sub>1</sub> 0467	Ed Standing still Students are working; Ed moves about with little involvement in any particular group; sits on a table and chats to a group occasionally.	Taking off their attitude that nothing is being done in Ed's lectures. Making a show of not meaning to offend these over-sensitive people and of not committing what he was accused of.
II Ed, Lecturer					Sincere attempt to heed criticism. Are they ignoring each other, students and Ed? Perhaps both feeling estranged.
II Ed, Lecturer		"You preferred this morning to last Monday morning?"	V <sub>1</sub> 1830	Ed. Still	He's feeling better about the work they have obviously being doing; he's asking for approval.
IV Action Research		(Ed refers to Committee's future task of reviewing this lesson and designing the next one).	V <sub>1</sub> 1842		The process is underway; improvement will continue.
	A <sub>1</sub> 11 370	END.	V <sub>1</sub> 1851		

ISSUE	RECORDING	INSIGHT
	<u>AUDIO</u>	
	<u>COUNTER</u>	
IV Action Research III Teaching	<p>A<sub>2</sub> I 016 Chris: (Trevor has pointed out Ed's habit of talking to one side of the class, as video is being viewed.) "These are the sort of comments we make..."</p> <p>A<sub>2</sub> I 092 Barbara: It was frustrating ... There were so many coming on and disappearing quickly. (She is remarking on comments about Ed's handling of transparencies.)</p> <p>A<sub>2</sub> I 098 Chris: You must thump him!</p> <p>243 Barbara: Just this one positive thing...</p> <p>255 Ed: I thne picked up there was a <u>need</u> for talk.</p> <p>A<sub>2</sub> 284 Lisa: I really was very frustrated.</p> <p>A<sub>2</sub> I 291 to 296 Chris: I've got a couple of questions.... How would you handle it?</p>	<p>She refers to Ed's short displays of transparencies leaving no time for reflection or comment by students. She shows irritation, sounds critical and <u>is</u> attacking Ed.</p> <p>We're being alerted to Ed's tactics; also implying that rolling up one's sleeves and getting down to it is going to mean we don't pull punches with Ed.</p> <p>Barbara in an encouraging, expansive role in contrast to criticism she has levelled at Ed previously.</p> <p>Lisa skilfully directs attention to herself - off Ed - and yet manages to criticise his teaching.</p> <p>Group teaching and question of groups wandering off the subject - desirability of it. The use of this process is appreciated as it is demonstrated. The topic of teaching serves its obvious function of informing us, but also Ed, while ostensibly off the hook, is still under spotlight.</p>
III Teaching	A <sub>2</sub> I 291 to 296 Chris: I've got a couple of questions... How would you handle it?	Chris mentions another of Ed's classes that I find him quite acceptable. So we're all in this together; not accusers versus the accused, students and Ed respectively. We're going to learn together - a collaborative venture.
IV Action Research III Teaching	<p>A<sub>2</sub> I 377 Chris: But Ed hasn't been under a lot of flak. There's another group saying we don't know what all the fuss is about.</p> <p>A<sub>2</sub> I 381 to 393 Chris: What you might be talking about as well, is a personality characteristic of Ed ... ... which are intended to close the gap between teacher and student ... but in actual fact are distancing...</p> <p>A<sub>2</sub> I 403 Trevor: You wanted to get that feeling of that you're teaching us and we're students away.</p> <p>A<sub>2</sub> I 411 Chris: It's interesting to see these type of things and this is why its valuable .....</p> <p>A<sub>2</sub> I 506 Chris: But it's subtle because it says ...</p>	<p>Just in case you got the wrong 'idea'. The above statement about Ed's acceptability was to help Ed but <u>not</u> to let him off the hook. He <u>is</u> under scrutiny.</p> <p>Valuable teaching points can be picked up in this sort of process.</p> <p>Merits and demerits of different worksheets are under discussion and the teaching/learning situation is foremost.</p>



ISSUE	AUDIO RECORDING	INSIGHT
<p>IV Action Research II Ed, Lecturer</p>	<p><u>COUNTER</u></p> <p>A, I 050 Ed: "As far as I <u>know</u>, Chris has a video that <u>is</u> to be shown."  Student: "How long is the video?"  Ed: "I haven't the <u>faintest</u> idea."  "The way I read it..."</p> <p>082 Barbara: "I think we need to know the content."  " So how are you going to do your lesson."  Ed: "I haven't the <u>faintest</u> idea."</p> <p>085 Trevor: "I think that's most exciting. You see..."</p> <p>112 I: "I think ... works."</p> <p>127 Student: "Why don't we try for you to do <u>no</u> talking?"</p> <p>130 Ed: "Do you realise the kind of torture you're placing me under?"  "Well, if <u>that's</u> the feeling of the group,"</p> <p>145 Ed: "Are you <u>saying</u>...?"</p>	<p>This whole business is not <u>my</u> fault; I've just been asked to do this lecturing; I'm representing Chris. See how little <u>I</u> have to do with it. (Chris not present.)</p> <p>He's approaching this whole process with 'lang tande'. Ed's affected, couldn't-care-less attitude indicates he doesn't appreciate what all the fuss is about; it's quite unnecessary, all these meetings, etc.</p> <p>Barbara expresses dismay, irritation of the students in the Committee.</p> <p><u>A Crisis</u>: Why don't we all pack up? Without Chris we aren't able to work.</p> <p><u>Crisis Averted</u>: The turning point here in this whole negotiation process.</p> <p>Co-operation!</p> <p>Humility and willingness to discuss his ways of teaching; Ed is heeding their suggestions.</p>

ISSUE	RECORDING		INSIGHT	
	<u>AUDIO</u>		<u>VIDEO</u>	
	<u>COUNTER</u>		<u>COUNTER</u>	
II. Ed, Lecturer	A <sub>4</sub> I 000 to 381	Ed: "I think you all know Sue... who is... Tom Vincent who has taken the role as the observer."  "Where the interrelationships between you - the student body - and myself... put under the spotlight." (Factual)  "What we are going to do..."  "I'd like you to break into groups..." (Confident)  "There are two principles on which I am working..." (Authoritative)  "I <u>suggest</u> you take down notes."		Confidence? Acceptance? I think so.
I Class II Ed, Lecturer	A <sub>4</sub> I 874	(A group can be heard in serious discussion on the topic of subtraction as in the video.)		The Action Research <u>is</u> operating; this lecture is a joint effort.  Quietly authoritative, without bombast.  He's confident, prepared.  Non-prescriptive.
	965	Ed: "Who would like to kick off...?" (Trevor responds; after that many students contribute, Ed quiet.)	V <sub>1</sub> 1854 Ed still , at front of class; composed.  1903  1923 (Video off; subtraction video being shown.)  1925 (Group discussions on the go.)  2167 Ed at front quietly composed; face relaxed.  Ed waits quietly for a group to accept offer to present their point of view.  Ed surveys students quietly, expectantly.	Ed is more at home with the class, with his lesson.  No touchiness about class's delay to respond.  Ed <u>is</u> listening, responding positively.  He is at ease, genuinely wanting students to respond, drawing them out by his prompts like "Do you feel/find ...  Rapport is excellent; debate constructive.
	A <sub>4</sub> II 000	Ed: "Do you feel...?" (Sincere)		
	006	"Do you find the teacher's behaviour intrusive?" (Inviting response) (Lisa affirms it) "Ed, why do you say that?"	2810	
	II 044	(Next subtraction video starts)		
	II 047	(Groups discuss video)	2895 Groups in discussion (Noticeable that Committee students predominate in discussions)  Ed's mannerisms subdued, if not absent completely.	
	II 141	(End of Lecture)	2986 Class starts to leave.	

ISSUE	RECORDING		INSIGHT
	<u>AUDIO</u>	<u>VIDEO</u>	
	<u>COUNTER</u>	<u>COUNTER</u>	
II Ed. Lecturer	1 002 Ed: "Would you people like to continue talking?"  030 Group discussions  064 (Tape Recorder off)	V <sub>1</sub> 3131 (Ed invites discussion on School Experience)  3169 (Video off)  3395 (A statement about language being divorced from the concrete has been made in the report-back session. Ed refers to the statement, adding): "and that's no criticism".  3772 (Ed addressing a student <u>asks</u> about adding in subtraction.)  4099 (Ed and Lisa - a student- engaged in discussion while groups are under way.)  End of V <sub>1</sub>  V <sub>2</sub> 0000 (Lecture continued, to ending on this tape) 0384	At ease. amenable to students' suggestions and accommodating their inclinations.    Ed is being supportive, encouraging student participation.  Ed solicits student opinion <u>and</u> listens to responses. Obviously he is confident if he is prepared to ask students.  There is interest being shown, first on the part of Ed which encourages concomitant attention from students.   Instead of shrugging people off, Ed now takes them into account.

ISSUE	VIDEO RECORDING	INSIGHT
Ed, Lecturer and Teaching	<p><u>COUNTER</u></p> <p>V<sub>2</sub> 0408 (Lecture of 19 May is being viewed.)</p> <p>0635 "In a class what would you do Ed?" (We are considering why students are reluctant to speak on their School Experience which is just past.)</p> <p>0814 Ed: "Tom, there's no way ..." (Ed points a finger, sounds irritated. Query is still about the students' refusal to talk in the class.) Tom: "That's defeatist."</p> <p>088 (Ed hears back to the familiar, to a role where his authority was accepted and largely unquestioned. He gives us an account of the <u>instant</u> response that a similar invitation to the college students would have produced. He lifts his head and shakes it vigorously, "... and this is <u>not</u> a criticism...")</p> <p>0973 Ed: "No, I hear you loud and clearly actually." (Hands between knees, relaxed.)</p> <p>1113 (Sue - the video operator - makes a suggestion as to how Ed might have put the invitation to the class.) Ed: "I have difficulty with that." (He responds to her suggestion. Hands relaxed and between knees.)</p>	<p>The Topic Teaching is serving its dual role. Ed is having to look at himself - an experience salutary for all or us - and we are reflecting on a real problem in teaching, hearing one another's comments about it.</p> <p>Ed is sensitive; Barbara just previously has criticised him about being in a hurry, and he feels he is in the dock. Lisa and Barbara are both questioning the response he had made earlier.</p> <p>I'm sorry for this 'defeatist' statement as soon as I've made it. Accusation brings about defensiveness and discussion take a back seat. I have been anything but a facilitator when I let out a remark like that! I am looking now for opportunity for making reparation.</p> <p>Ed is annoyed; he is reprimanding those students present by comparing them unfavourably with those at college. Of course, this is just what it is : criticism.</p> <p>Ed is in total agreement because I have just made up for my mistake, by speaking of the responsible role he fills at College. Ed is disarmed.</p> <p>That our video operator should enter into the discussion indicates the interest being engendered about the teaching point in question.</p> <p>Ed is back in the discursive mood; he is relaxed and answers to the teaching problem are once more occupying us all.</p>

EXPLORATIONS IN TAILORING AND IMPLEMENTING  
MATHS LESSONS THAT AIM FOR CHANGE

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The Marine Primary School in the Ocean View township took me by surprise. A depressed socio-economic community, like that at Ocean View, usually had low standards, a deprived outlook and poor aspirations, reflected invariably by rough behaviour, poor discipline and dirty condition of the pupils at its schools. Marine Primary was the complete opposite. It had neat fields and gardens, spotless buildings - even if old - a dignified principal and staff, and well-behaved children. Children and staff seemed proud of their school and the will to learn on the part of the children and desire to teach on the part of the staff was evidence of their desire to be part of it all.

This aggravated my misgivings about Ken Dovey's project. My reservations about applying a theory for a liberating education, basically the aim of the project, assigned to the M.Ed. students by Ken Dovey after his lectures to us, were intensified to the degree of downright opposition. Where was the need to 'liberate' this happy community? And yet there was the lingering question: Why did an opportunity to extend freedom to these people, who were classed as coloured, banished to this apartheid-inspired village as far from the surrounding white villages as possible, seem undesirable? It seemed that arousing 'critical awareness' in this contented school would be causing an unwanted disturbance.

I needed to return to reading Ken's notes (Dovey, 1986) and the references he left us. First of all, Ken's references to schools he had investigated in the search for symptoms of oppression amongst pupils and staff tallied, surprisingly, with Marine Primary, the school in question. Teachers - enthusiastic and diligent as they were - were subject to a strict merit award system, were spoon-feeding the pupils and were subjecting them to a discipline that exercised control typical of the positivist pedagogical model. This model, according to Giroux (1981, p.56) does not encourage pupils "... to generate their own meanings, to capitalize on their own cultural capital, or to participate in evaluating their own classroom experience."

Staff and pupils did accept quite uncritically the situation that Giroux describes as follows : "When he enters the classroom for the first time the sense of order and meaning are not a matter of negotiation, between himself, his peers and the teacher." (1981, p.22).

C.A. Bowers, referring to Paulo Freire's philosophy, writes, "In order for the individual to change himself he must be able to free his consciousness from the conditions that shape it." (1974, p.10).

Freire's idea that change "involves a rapid movement in search of new themes and new tasks" (1973, p.7) demands that "man needs more than

ever to be integrated with his reality" (1973, p.8). This places education in the forefront as an agent of change, Freire considering it as the force that affords man the awareness of reality, enabling him to seek radical solutions, ally himself with them and be one who "does not deny another man's right to choose", nor "impose his own choice" (1973, p.10). Such a man is capable of discussing the respective positions of others, and while "convinced he is right", respecting others' judgments, and occupying essentially a "predominantly critical, loving, humble and communicative, and ... positive stance" (1973, p.10).

In contrast, people like the majority of Brazilians, described by Freire in his reference to the 'transitional phase' in Brazil's history (1973, p.8), were subjects of a closed society, caught up by the change instead of working for it. The "particular meaning and emphasis given by a closed society to themes like democracy, popular participation, freedom, property, authority and education were no longer adequate..." (Freire, 1973, p.8).

Thus for Freire, where violent political change is often superficial and merely the substitution of one tyranny for another, education can bring about democracy by establishing it as "a form of life ... before it becomes a political form" (1973, p.29). It is not just to the leaders and other prominent figures that education, as the indispensable factor of change, is to be offered, but to everyone. It was to the majority of Brazilians who lacked the education, that

could have made them more than 'mere spectators' (1973, p.13), because, according to Freire, lack of education had left them "unprepared to evaluate the transition critically". (1973, p.11).

If noble aspirations were the only incentive for the institution of long term programmes of upliftment they would be justification enough for agitation for immediate commitment to a radical education system in South Africa. But we stand at the brink of catastrophe in this country. Our politics is about survival and involves all the inhabitants. Privileged - mainly white people - and oppressed - the coloured and black peoples - need corporately to usher in a new dispensation of justice for all. We all need to be aware of the issues involved in the change - those that need discarding and the others and new ones that we need to adopt.

I include all South Africans because the white people, like the elite in Brazil, are also unprepared, and consequently prone to assume 'sectarian' stances that neglect the "vigilance of reflection" (Freire, 1973, p.11) and remove a capacity for love and the ability to tolerate others' opinions and choices. An education that informs and prepares will be necessary for them also. Young urban black people, and some coloured youths, are suffering the destructive effects of chaotic conditions in schooling, but in the process are perceiving more than their white counterparts what the issues at stake are. The licence they have assumed has allowed a certain freedom to discuss, reflect and take combined but limited action that affords them experience of power politics, denied white youths.



Many black schools in country districts and most coloured schools carry on schooling along the lines of the white schools, however, suffering the same deprivation of awareness. Marine Primary is no exception. Bowers' description of the difficulties confronting a pupil in the schooling he writes about, is applicable to ours : "The task of sorting out his own responses to different aspects of the culture is difficult enough, especially when he must also take into account how other people interpret it, but it is made even more difficult by the existence of school routines, the competitive atmosphere of the classroom, and the model of the good student that is continually being presented for him to emulate." (1974, p.24)

Do we have to look outside the schools, then? Stanley Aronowitz, in his preface to Giroux's book, writes that Giroux does regard schools as agents of change, saying of Giroux, "... he leaves room for the possibility that the conditions of learning - the classroom, textbooks and other spaces where people try to gain power through understanding - may be counter-hegemonic." (Giroux, 1981).

However, he does not see it taking place by default, stating that "... classroom knowledge ... can be used unreflectingly to legitimize specific socio-political interests by appearing to be value free and beyond criticism." (Giroux, 1981, p.54).

Teachers at schools - at Marine Primary, for example - have to consciously work for change, actively combating the tendency to accept the constraints about them, acting as "strong influences without being 'superiors', controlling learning totally", according to Freire (Frankenstein, p.15).

The question is: Can the cultivation of critical awareness of one's world - in the case of teachers and pupils, their school and classroom constraints - start where one is at? Or do massive movements, involving teacher-training programmes and in-service courses at colleges and universities, have to be instituted initially? Paulo Freire, Giroux, Hull and Marilyn Frankenstein are among many who appreciate and press for the efforts of individuals in their teaching situations in the belief that they can work towards change. Ken Dovey's visit to us, his lectures to the students in the M.Ed. group and this project he has set for us are stimuli to us because they testify to his belief in the power of the individual teacher as an agent of change.

Charles Hull recommends the critical examination of the accepted constraints of the classroom. Such a "discourse together about classroom processes" (Hull, 1985, p.6) refers to discussion between the teacher and his pupils. In my lesson to the Std 5 class, I made provision for a tape recorder, which was to - in a very small way - initiate pupils into a process whereby "the mechanisms of the hidden

curriculum, conventionally realised through forms of teacher-pupil interaction that are themselves the blueprints of the schooling are exposed to examination by participants." (Hull, 1985, p.6). I hoped that "... through such manipulation participants can grasp the situations they find themselves in and turn them to their own ends" (Hull, 1986, p.6), intending that the recording of a group's discussions would be replayed, listened to together and discussed.

Maths teachers, on the whole, are left unmoved by Freire's plea for teachers "to explore the non-positivist nature of the knowledge we are teaching" (Frankenstein, pp.15/16). I had never regarded maths as anything but a value free, a-political subject, until Ken Dovey, in one of his lectures (1986), described maths as "a tool for understanding and reconstructing a new world". Further reading led me to regard maths as serving man in two ways:

- (i) as a being living in his world, and

- (ii) as a being possessed of tremendous potential for thinking.

Maths took on new power for me; I could see it serving, foremost among other subjects, the educational process of "making problematic what one has grown up with" (Dovey, 1986) and developing skills that enable man's using maths to effect change.

When one regards maths as a language and applies arguments by Freire for his literacy process to maths, it assumes great power as an agent of radical education. It enables one to exercise the freedom to name the world - "a creative and transformation process", according to

Dovey, who added that "a literate person is immune to political oppression" (1986). As Freire believes, "... the literary process must relate speaking the word to transforming reality and to man's role in this transformation" (1970, p.213), one appreciates the numeracy process's role of relating the manipulation of number to a transformation of reality. Before this can take place, however, skill with number needs to be developed.

My two lessons, one to a Std 3 class, the other to Std 5's, by accident, followed the two steps in order. The lesson on making and using dice represented the acquiring of number skills while the second lesson involved the Std 5's in the role of number in their world where they considered and calculated sales tax. Both lessons were departures from the preoccupation with textbooks - a practice at its worst among maths teachers. Freire deplores the use of them in teaching literacy, maintaining : "Intellectualist prejudices and above all class prejudices are responsible for the naive and unfounded notions that the people cannot write their own texts, or that a tape of their conversation is valueless since their conversations are impoverished of meaning." (1970, p.225).

The parallel with the use of maths textbooks is striking, reflecting how little store maths teachers put on the maths knowledge that children have themselves. My lesson on sales tax aimed at utilizing the children's experience of shopping as the basis on which to investigate taxes, percentage and calculations with these.

I aimed to have children using their methods of calculating tax by leaving them to answer questions about the sales tax on the articles they had brought to school. No instruction about percentage or the calculation of tax was given beforehand. To some small degree here, I tried to avoid what Freire deplores as the words being "deposited" instead of "being born of the creative effort of the learners", adding, of the study of so-called reading lessons, that they are "almost completely alienating and alienated, having so little, if anything, to do with the student's socio-cultural reality". (1970, p.208).

The children in the Std. 5 lesson were arranged in groups in an effort to change them from the role of passive recipients to that of co-investigators. I wanted them to share their experiences with one another and by social contact reduce tension that could inhibit thinking. There was scope in the worksheet for thinking about operations with numbers and for thoughts that examine maths' place in the world they inhabit. I acted as listener and facilitator, the tape recorder being placed with one group for a record of the discussion of the group. The first stage was to make children aware of the sales tax and to help them develop calculating skills around it.

The second stage had children in groups discussing the merits of tax, its effects on their lives and the fairness of it and necessity for it. Hopefully discussion on these points would help a critical

awareness of a factor in their lives to take root in these children.

The introduction of some common simple article, as a source of mathematical activity, to the children was my aim in the dice-making lesson to Std 3's. The cultivation of the thinking associated with posing problems and finding solutions would, I hoped, develop from children's fascination with dice and the games they could play with them. I find maths anxiety an ever prevalent phenomenon, occurring in varying degrees in most children, affecting those it attacks to the point that any mathematical thinking is threatened with paralysis - "my mind went blank". Bringing milk cartons from their homes, constructing dice and the talk about numbers that ensued was to play a major part in reducing maths anxiety.

After constructing the dice in the first lesson, the subsequent one saw the Std 3's thinking about their use of them. Pupils arranged themselves in groups, and after some demonstration of games using the dice, pupils were left to develop on these games. Constructing the dice had sufficiently 'loosened up' the class to the extent that they were ready to venture into activities with numbers.

How the dice are used, of course, determines whether real critical reflection is encouraged and whether the dialogue, questioning and communication is meaningful. Nevertheless, having at hand what one has made - a means of supplying numbers for oneself - can be regarded as an investing of pupils with power which textbook exercises do not afford. A start at least was being made towards mathematizing.

The unqualified delight with which the children produced endless numbers for a variety of games and other activities indicates that some sort of liberatory process was operating. Implicit in the activity for the pupils is that they are active proponents in the mathematical process.

Grouping children as was arranged in both the lessons encourages a process that wrests control out of the teacher's hands, combating the principles of "order, control and certainty" that normally prevail and that "appear inherently opposed to student development". (Giroux, 1981, p.56). For a start, group work implies that dialogue is no more between a teacher and the class as a unit, and the activity and discussion in groups are freer than those in traditional situations, where all comment and queries have to pass by way of the teacher. The teacher is 'unpinned', as it were, from his or her position at the front of the class and from the role of a 'central processing unit'.

The lesson on tax was an attempt to move away from the influence of the textbook and a view of education where, as Giroux states, "... knowledge is often treated as an external body of information, the production of which appears to be independent of human beings. From this perspective, knowledge is viewed as independent of time and place; it becomes universalised, a-historical knowledge." (1981, pp.52/53). Once children appreciate that tax is part of their world, in its form as sales tax, questions are posed that encourage them to reflect on the extent of it, the amount of it and the means of levying tax.

## AFTERTHOUGHTS

As essays into the encouragement of thinking in maths, the lessons described have served only as introductions. Had my dice lesson sensitised children to number on their lives because we used materials - paper and cartons - typical of their world for mathematical purposes? I think so. The children played with the dice long after the lessons when we first used them, bringing them out when other work was completed, and continuing games that had been introduced.

How much, however, besides what novelty, initial excitement and experiment can supply, can be learnt about mathematizing unless careful thought is expended beforehand on the concepts involved and the ways that dice can help establish them? I feel the die sets the scene from where investigations can proceed, that can set children to a fearless seeking, and to enthusiastic attempts to find patterns and properties belonging to the world of number. Devices, like dice, have the knack of inducing spot investigations that can be applied at any stage of maths, switching one into a mathematizing mode whatever the topic. I think the lessons demonstrated this. However, I did not attempt to devise structured investigations that - while allowing pupils to experiment - did have them following certain paths where planned-for properties and patterns could be happened upon. This should be the next step, working from the gains achieved, particularly in attitude.

The Std 5 lesson on tax had tape that gave me some ideas about the outcome of the lesson. (This tape is included as part of this



project.) Initially, I introduce the group activity idea to the children, although they affirmed they had done group work. Where the counter indicated 109, the tape recorder was placed with one group. To hear amongst the buzz, representing conversation on maths, a question from a boy like, "How do you work out the tax?", being asked of the rest of his group, was, I felt, an improvement on my asking one pupil in the class. How many other questions of that nature were not being asked simultaneously, was an encouraging speculation.

However, the question was imposed in that it appeared on the work card, issued to each group. Ideally questions need to be posed by the pupils themselves, and starting perhaps with scenes of shop-owners and customers, selling and buying and calculating sales tax, might highlight more effectively facts about the tax and induce questions on the part of pupils themselves? My role as facilitator succeeded, in that in the report-back session, when groups presented their calculations of tax, their methods were examined and discussed. My encouragement of the use of children's methods instilled confidence in them about their abilities. Their assessment of methods was taken into consideration by - as much as possible - my refraining from responding with 'good' and 'right' to their answers. Rather, I responded by asking of the class, "Do you agree?"

I found sales tax a suitable topic to make children aware of their world, experiencing at the same time how some children - a few in each group - were mathematizing to the degree that various attempts were being made by them to arrive at answers. In the course of these

attempts, questions were being asked of one another, answers were being discussed and accepted or dismissed, and this all took place in a relaxed atmosphere. How many children were deterred from trying much because 'tax' and 'per cent' were unknowns to them, I am not sure about. It makes me hesitate to rush into relating 'problems' to the currency of the context too avidly because one can sometimes alienate pupils in this way. Simple common materials can leave problems 'unattached', keeping them neutral and acceptable to all children and therefore more effective.

The use of the tape recorder to examine classroom practice was unfortunately missed, because the time was up and I could not get to that class again, as also was the planned discussion about the need for tax and the justice of it. Such is inevitable when one is a visitor, and regrettably the intended positive effects of the investigation on the children involved are achieved only to a small degree or not at all. Enough evidence in support of one's premise, however, should be able to be gained, so that conclusions about what is possible can be made. This excursion into the exciting of critical consciousness in children at Marine Primary has seen theory being applied in however small a way. It has nurtured my growing convictions about the power invested in the individual teacher for bringing about change.

## REFERENCES

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